FINAL EXAMINATION

MATH 201

January 24, 2008; 11:30 A.M.-1:30 P.M.

Name: Signature:

Circle Your Section Number:

17. 18 19 20

24 25

Instructions:

- There are two types of questions:

PART I consists of six work-out problems. Give detailed solutions.

PART II consists of eight multiple-choice questions each with **exactly one correct answer**. Circle the appropriate answer.

Grading policy:

- 10 points for each problem of **PART I**.
- 5 points for each problem of **PART II**.
- 0 point for no, wrong, or more than one answer of **PART II**.

GRADE OF PART I	/60
GRADE OF PART II	/40
TOTAL GRADE	/100

Part I (1). Use Lagrange Multipliers to find the absolute maximum and minimum values for the function $f(x, y, z) = x - y + z$ on the unit sphere $x^2 + y^2 + z^2 - 1 = 0$.

Part I (2). Use cylindrical coordinates to find the volume of the solid bounded above by the surface $z = e^{\sqrt{x^2+y^2}}$, below by the $xy-$plane, and laterally by the cylinder $x^2 + y^2 = 1$.

Part I (3). Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + 3y - 3xy$ over the triangular region R bounded by the $x-$axis, the $y-$axis, and the line $x + y = 6$.

Part I (4). Evaluate the integral

$$\iint_R \frac{2y + x}{y - 2x}\, dA,$$

where R is the trapezoid with vertices $(-1, 0)$, $(-2, 0)$, $(0, 4)$, and $(0, 2)$, by using the substitution $u = y - 2x$ and $v = 2y + x$.

Part I (5). Integrate the function $f(x,y)=\sqrt{x^2+y^2}$ over the parametric curve $C:\ \ \vec{\mathbf{r}}(t)=(\cos t+t\sin t)\,\vec{\mathbf{i}}+(\sin t-t\cos t)\,\vec{\mathbf{j}},\quad 0\le t\le\sqrt{3}.$

Part I (6). Sketch the region of integration of the integral

$$\int_0^4 \int_{\sqrt{y}}^2 x^3 \cos(xy)\, dx\, dy,$$

and evaluate the integral by reversing its order of integration.

Part II (1). If $f(x, y)$ satisfies Laplace's equation $f_{xx} + f_{yy} = 0$, then the value of the integral $\int_C f_y dx - f_x dy$ over all simple closed curves C to which Green's theorem applies is

(a) 0.

(b) 1.

(c) -1.

(d) π.

(e) $-\pi$.

Part II (2). The mass of the annular metal plate bounded by the circles $r = 1$ and $r = 2$ and whose density function is $\delta(r, \theta) = \cos^2 \theta$ is

(a) $\pi/4$.

(b) $\pi/2$.

(c) $3\pi/4$.

(d) $3\pi/2$.

(e) None of the above.

Part II (3). The volume of the solid lying below the half-cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 8$ is given by the triple integral

(a) $\int_0^{2\pi} \int_{\pi/4}^{\pi} \int_0^{2\sqrt{2}} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

(b) $\int_0^{2\pi} \int_{\pi/3}^{\pi/4} \int_0^{2\sqrt{2}} \rho \sin^2\phi \, d\rho \, d\phi \, d\theta$.

(c) $\int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

(d) $\int_0^{\pi} \int_{\pi/4}^{\pi} \int_0^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

(e) None of the above.

Part II (4). If

$$a_n = \begin{cases} 1/\sqrt{n}; & 1 \leq n \leq 100 \\ (-1)^n/n; & n > 100, \end{cases}$$

then the series $\sum_{n=1}^{\infty} a_n$ is

(a) convergent.

(b) absolutely convergent.

(c) conditionally convergent.

(d) divergent.

(e) None of the above.

Part II (5). The power series

$$\sum_{k=2}^{\infty}(-1)^k \frac{(2x-1)^k}{k6^k}$$

has interval of **absolute** convergence

(a) $-5/2 < x \leq 7/2$.

(b) $-5/2 < x < 7/2$.

(c) $-5/2 \leq x < 7/2$.

(d) $-5/2 \leq x \leq 7/2$.

(e) None of the above.

Part II (6). The directional derivative of the function

$$f(x, y, z) = x^2 + 4y^2 - 9z^2$$

at the point $P(3, 0, -4)$ in the direction from P to the origin is

(a) 52.

(b) -52.

(c) 54.

(d) -54.

(e) None of the above.

Part II (7) The function $f(x,y) = x^2 - 4xy + y^3 + 4y$

(a) has local minimum at $(4, 2)$ and local maximum at $(4/3, 2/3)$.

(b) has local minimum at $(4, 2)$ and saddle point at $(4/3, 2/3)$.

(c) has local maximum at $(4, 2)$ and saddle point at $(4/3, 2/3)$.

(d) has saddle point at $(4, 2)$ and local minimum at $(4/3, 2/3)$.

(e) None of the above.

Part II (8) An equation of the tangent plane to the surface $2x^2 - y + 5z^2 = 0$ at the point $(1, 7, 1)$ is

(a) $3x - y + z = -3$.

(b) $x + y + 2z = 10$.

(c) $10x - y + 4z = 7$.

(d) $4x - y + 10z = 7$.

(e) None of the above.