Please note that you have 9 questions and 9 pages

1) **(13 points)** Consider the curve $f(x) = -x^4 + 2x^2 + 1$.

a) Find the equation of the tangent line to $f(x)$ at $x = 2$.

b) Does $f(x)$ have any horizontal tangent? If yes, say where. If no, say why.

a) $f(2) = -(2)^4 + 2(4) + 1$

$= -16 + 8 + 1 = -7.$

$y + 7 = a(x - 2)$

$y = ax - 2a - 7$

$a = f'(2)$ so $f'(x) = -4x^3 + 4x$

$f'(2) = -4(2)^3 + 4(2)$

$= -4(8) + 8$

$= -32 + 8 = -24.$

$a = -24$

~~$y = -24x - 2(-24)$~~

$y = -24x - 2(-24) - 7$

$= -24x + 48 - 7$

$= -24x + 41$

b)

2) (10 points) Find the derivative of each of the following functions:

a) $f(x) = \dfrac{1-x}{1+x}$

$$f'(x) = \frac{-1(1+x) - 1(1-x)}{(1+x)^2} = \frac{-1 - x - 1 + x}{(1+x^2)} = -\frac{2}{(1+x^2)}$$

10

b) $f(x) = x^3 - \dfrac{1}{x} + 2\sqrt{x}$

$x^{1/2 - 1}$ $\quad x^{-1/2}$ $\quad \frac{2}{\sqrt{x}}$

$$f'(x) = 3x^2 - \frac{0(x) - 1(1)}{x^2} + \frac{1}{2}\left(\frac{2}{\sqrt{x}}\right)$$

$$= 3x^2 + \frac{1}{x^2} + \frac{1}{\sqrt{x}}$$

3) (8 points) Given $f(x)=\dfrac{x^3-2x}{x^2+1}$.

Show whether $f(x)$ is odd, even function or neither.

$f(-x)=\dfrac{-x^3+2x}{x^2+1}=-f(x)$ odd. $f(-x)=-f(x)$

8

4) **(8 points)** Given $f(x) = 4x - 1$ and $g(x) = (x+1)^2$.

a) Find $f \circ g(x)$.

$$f((x+1)^2) = 4(x+1)^2 - 1$$
$$= 4(x^2 + 2x + 1) - 1$$
$$= 4x^2 + 8x + 4 - 1$$
$$= 4x^2 + 8x + 3.$$

8

b) Find $g \circ f(x)$.

$$g(4x-1) = [(4x-1)+1]^2.$$
$$= (4x)^2$$
$$= 16x^2.$$

$x^2 + y^2 = 1$

5) **(13 points)** Given the parametric equations,

$x = -3\cos t$ and $y = 2\sin t$ for $\frac{-\pi}{2} \le t \le \frac{\pi}{2}$.

Find the Cartesian equation for the given parametrization, then graph it indicating the initial point, terminal point, and the direction of the motion.

$\begin{cases} x = -3\cos t \\ y = 2\sin t \end{cases}$

$9\cos^2 t + 4\sin^2 t = 1$ it's an equation of a circle.

$\frac{1+x^2}{1-x} \sim \frac{x^2}{-x}$ $\frac{1}{x}$

6) **(10 points)** Let $f(x)=\frac{1+x^2}{1-x}$.

Find the asymptotes of $f(x)$, if any.

$$f(x) = \frac{1+x^2}{1-x}$$

$1-x \neq 0$

$-x \neq -1$

$x \neq 1$

$D_f =]-\infty; 1[\cup]1; \infty[$

$$* \lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x^2}{-x} = \lim_{x\to-\infty} \frac{x}{-1} = \infty$$

$$a = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{1+x^2}{x-x^2} = -1$$

$$b = \lim_{x\to-\infty} f(x) - ax = \lim_{x\to-\infty} f(x) + x = \lim_{x\to-\infty} \frac{1+x^2+x(1-x)}{1-x}$$

$$= \lim_{x\to-\infty} \frac{1+x^2+x-x^2}{1-x} = \lim_{x\to-\infty} \frac{x}{-x} = -1$$

$\boxed{OA: y = -x-1}$

10

$$* \lim_{x\to1^-} f(x) = \lim_{x\to1^-} \frac{1+x^2}{1-x} = \lim_{x\to1^-} \frac{2}{0^-} = \infty$$

$\boxed{x = 1 \text{ V.A}}$

$$* \lim_{x\to1^+} f(x) = \infty$$

$$* \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2}{-x} = \lim_{x\to\infty} \frac{x}{-1} = -\infty$$

$\boxed{OA / y = -x-1}$

7) (18 points) Given the function $f(x)=\begin{cases} x^2 & x<0 \\ -x^2 & x>0 \end{cases}$

a) Graph $f(x)$.

b) Does $\lim_{x\to 0} f(x)$ exist? If no, say why. If yes, find the limit.

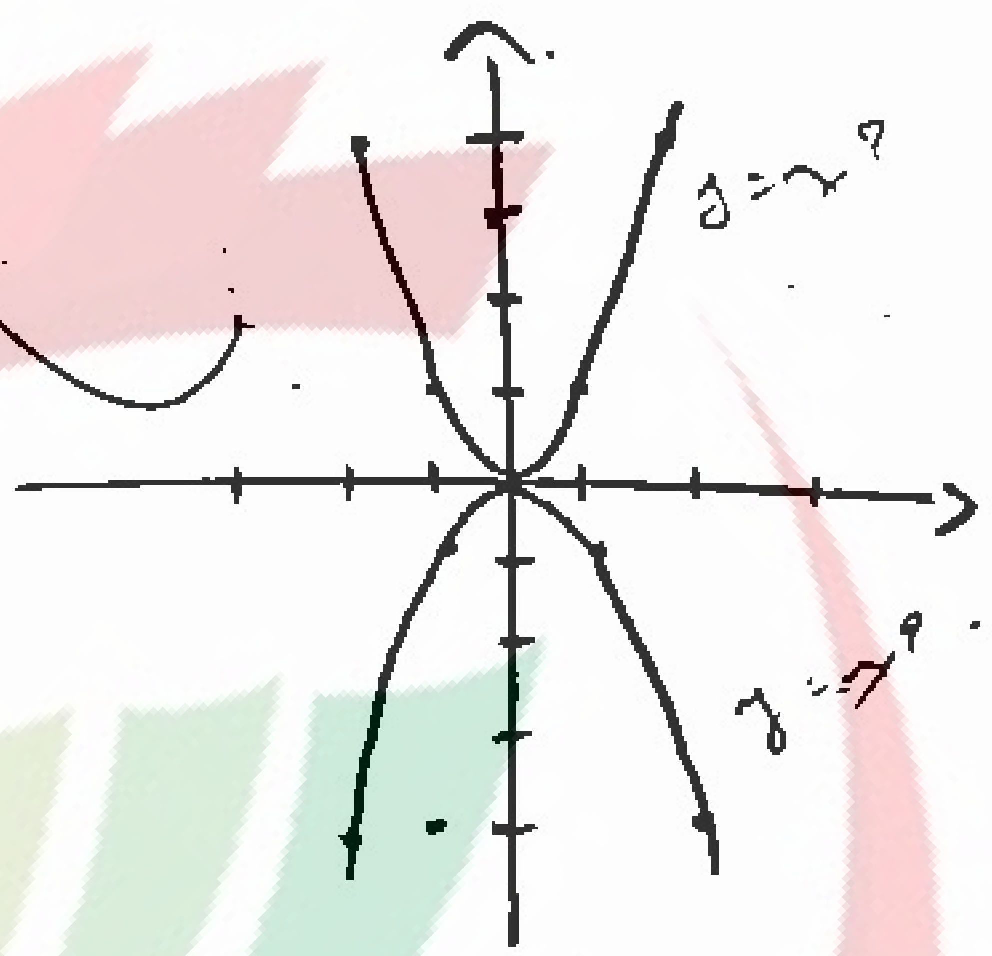
c) Is $f(x)$ continuous at $x=0$? Explain.

d) If $f(x)$ differentiable at $x=0$? Explain.

e) Can we extend $f(x)$ to be a continuous function over $(-\infty,\infty)$? Explain.

15

a)



b) yes because it belongs to Df.

$\lim_{x\to 0} f(x) = \pm x^2$.

c) No since $f(0^+) \neq f(0^-)$

d) Yes since $f(0^+) \neq f(0)$

e) yes but excluding 0.

8) **(10 points)** Consider the function $f(x) = x^5 - 3x^2 - 5x + 2$. Show that there exists at least one solution (root) of the equation $f(x) = 0$ in the interval $[0,1]$. (Do not try to find the root).

$f(x) = x^5 - 3x^2 - 5x + 2.$

$f'(x) = 5x^4 - 6x - 5$

$5x^4 - 6x - 5 = 0$

$x(5x^3 - 6) = 5$

$x = \frac{5}{5x^3 - 6}$

$f''(x) = 20x^3 - 6 = 0.$

$f(x) = 0$

$x^5 - 3x^2 - 5x + 2 = 0$

$x(x^4 - 3x - 5) = -2$

$\boxed{x = -2}$

$x^4 - 3x - 5 = -2.$

$x^4 - 3x = 3$

$x(x^3 - 3) = 3$

$\boxed{x = 3}.$

$x^3 - 3 = 3$

$x^3 = 0$

$\boxed{x = 0}$

$y = \sqrt{x}$

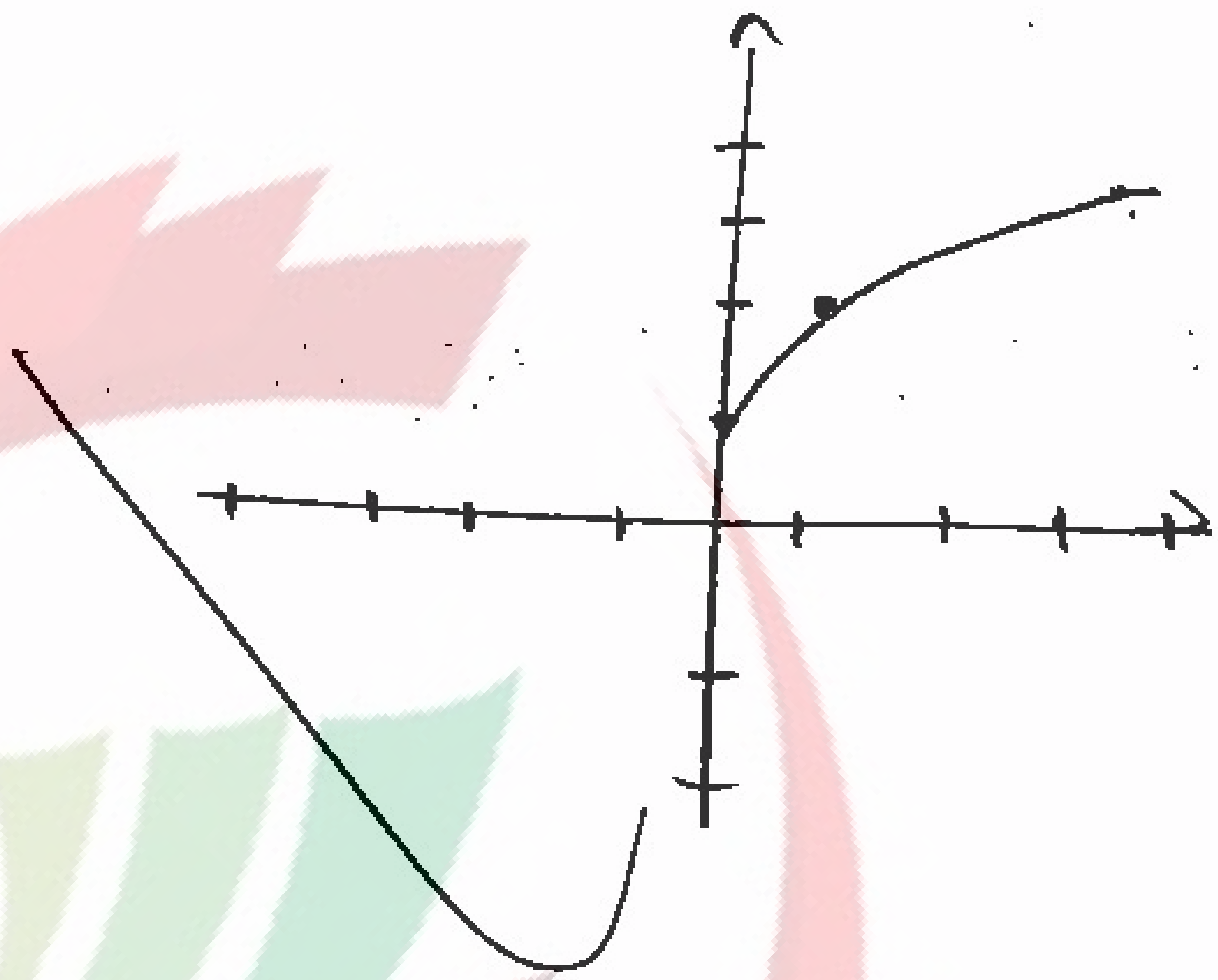
$y^2 = x$

$x = \sqrt{y}$

9) (10 points) Consider $f(x) = \sqrt{x} + 1$ for $x \geq 0$.

a) Graph $f(x)$.

x	0	1	4
y	1	2	3



b) Does $f(x)$ have an inverse function $f^{-1}(x)$? If yes, Graph it, then find $f^{-1}(x)$ specifying the domain and the range. If no, say why.

yes because its monotone on $[0; \infty[$.

$y = \sqrt{x} + 1$ so $\sqrt{x} = y - 1$

$x = (y-1)^2$.

$f^{-1}_{(x)} = (x-1)^2$.

$= (x^2 - 2x + 1)$