**American University of Beirut**

**MATH 201**

*Calculus and Analytic Geometry III*

*Fall 2007-2008*

*Final Exam*

Name: ..............................  ID #: ................................

**Exercise 1** *(12 points)* Discuss the convergence of the following series:

$$a)\ \sum_{n=0}^{+\infty} \frac{1}{\sqrt{n!}} \qquad b)\ \sum_{n=0}^{+\infty} \frac{n+10}{n \ln^3 n} \qquad c)\ \sum_{n=0}^{+\infty} (-1)^{n^2} \frac{1}{n^2+\sqrt{n}}$$

**Exercise 2** *(10 points)* If $w = f(x, y)$ is differentiable and $x = r + s, y = r - s$, show that

$$\frac{\partial w}{\partial r} \times \frac{\partial w}{\partial s} = \left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2$$

**Exercise 3** *(15 points)* Find the absolute extrema of $f(x, y) = 5 + 4x - 2x^2 + 3y - y^2$ on the region $R$ bounded by the lines $y = 2, y = x$, and $y = -x$.

**Exercise 4** *(8 points)* Use Lagrange multipliers to find the maximum and minimum of $f(x, y) = 4xy$ subject to $x^2 + y^2 = 8$.

**Exercise 5** *(15 points)* Evaluate the integral $\displaystyle\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin(2z)}{4-z}\, dydzdx$

**Exercise 6** *(10 points)*
Evaluate the integral $\displaystyle\int\int_D \cos(x^2+y^2)dA$, where $D = \{(x, y) \in \mathbb{R}^2; 1 \leq x^2 + y^2 \leq 2, y \geq 0\}$

**Exercise 7** *(20 points)* Let $V$ be the volume of the region $D$ that is bounded form below by the plane $z = 0$, from above by the sphere $x^2 + y^2 + z^2 = 4$ and on the sides by the cylinder $x^2 + y^2 = 1$

**a.** express $V$ as an iterated triple integral cartesian coordinates in the order $dzdydx$

**b.** express $V$ as an iterated triple integral cartesian coordinates in the order $dydzdx$

**c.** express $V$ as an iterated triple integral spherical coordinates

**d.** express $V$ as an iterated triple integral cylindrical coordinates, then evaluate the resulting integral.

**Exercise 8** *(10 points)* Set up an integral in rectangular coordinates equivalent to the integral

$$\int_0^{\pi/2} \int_1^{\sqrt{3}} \int_1^{\sqrt{4-r^2}} r^3(\sin\theta\cos\theta)z^2 dzdrd\theta$$

*(do not evaluate the integral)*

*good luck*