Vector & Tensor Analysis
Final Exam (two hours)

4 1) The derivative of $f(x,y)$ at $p_0(1,2)$ in the direction of $\vec{A} = \vec{i} + \vec{j}$ is $2\sqrt{2}$ and in the direction of $\vec{B} = -2\vec{j}$ is -3. What is the derivative of f in the direction of $\vec{c} = -\vec{i} - 2\vec{j}$?

(10 points)

10 2) If the rectangular cartesian coordinates system is rotated upward by 30° about the x_2-axis, a new system called $ox_1'\,x_2'\,x_3'$ is obtained. What are the components of $\vec{A} = (1,1,1)$ in the new coordinates system?

(10 points)

3) A vector field $\vec{F}$ is expressed in spherical coordinates system by $\vec{F} = F_\rho \vec{e}_\rho + F_\varphi \vec{e}_\varphi + F_\vartheta \vec{e}_\vartheta$.

05

a- Write the formula expressing $div\vec{F}$.

b- Find an explicit formula for the function $f(\rho)$ if $\vec{F} = f(\rho)\vec{e}_\rho$ and $div\vec{F} = 0$.

(10 points)

10 4) Let S denote the portion of the paraboloid $z = x^2 + y^2$ located between the planes $z = 0$ and $z = 4$. Find the area of S.

(15 points)

5 5) Evaluate the work of $\vec{F} = (e^{-y} - ze^{-x})\vec{i} + (e^{-z} - xe^{-y})\vec{j} + (e^{-x} - ye^{-z})\vec{k}$ along the path C parameterized by

$$r(t) = \left(\frac{1}{\ln(2)}\ln(1+t)\ ,\ \sin\frac{\pi}{2}t\ ,\ \frac{1-e^t}{1-e}\right) \qquad 0 \le t \le 1$$

(15 points)

15 6) Let $\vec{F} = (y^2 + z^2)\vec{i} + (x^2 + z^2)\vec{j} + (x^2 + y^2)\vec{k}$ and let C denote the boundary of the triangle cut from the plane $x + y + z = 1$ by the 1st octant.

If C is oriented counterclockwise, what is the circulation of $\vec{F}$ around C?

(15 points)

7) Let $\vec{F} = \frac{x}{(x^2+y^2+z^2)^{3/2}}\vec{i} + \frac{y}{(x^2+y^2+z^2)^{3/2}}\vec{j} + \frac{z}{(x^2+y^2+z^2)^{3/2}}\vec{k}$.

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Find the flux of $\vec{F}$ across the boundary of the region D described by $(x,y,z) \in D$ iff $a^2 \le x^2 + y^2 + z^2 \le b^2$.

(15 points) 0

$D_u f = (a,b)\frac{(1,1)}{\sqrt{2}} = 2\sqrt{2}$

$\frac{a}{\sqrt{2}} + \frac{b}{\sqrt{2}} = 2\sqrt{2}$

8) If A is a 2nd order antisymmetrical tensor with components a_{jk} and the vector $\bar{b}$ is given by

$$b_i = \frac{1}{2}\varepsilon_{ijk} a_{jk} \qquad i = 1,\ 2,\ 3.$$

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Show that $a_{rs} = \varepsilon_{irs} b_i$ and compute a_{11}, a_{12}, a_{13} in terms of b_1, b_2, b_3.

(10 points)