FINAL EXAMINATION

MATH 201

January 24, 2008, 11:30-1:30pm

Name:

Student number:

Section number (Encircle): 21 22 23

Instructions:

- No calculators are allowed.
- There are two types of questions:

Part I consists of ten multiple choice questions out of 5 points each with exactly one correct answer.

Part II consists of five work-out problems out of 10 points each. Give a detailed solution for each of these problems.

Part I:

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1 - The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{nlnn}$:

a. Converges absolutely

b. Converges conditionally

c. Diverges

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2. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+\pi)^n}{\sqrt{n}}$ is:

a. $]-1-\pi, 1-\pi[$

b.$]-1-\pi, 1-\pi]$

c. $[-1-\pi, 1-\pi[$

d. $[-1-\pi, 1-\pi]$

e. None of the above

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3. The Maclaurin series of the function $\int_1^x \frac{ln(1-t)}{t} dt$ is:

a. $-\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)^2}$

b. $-\sum_{n=1}^{\infty} \frac{x^{n+1}}{(n+1)^2}$

c. $-\sum_{n=0}^{\infty} \frac{x^{n}}{n(n+1)}$

d. $-\sum_{n=1}^{\infty} \frac{x^{n+1}}{n(n+1)}$

e. None of the above.

4. The function $f(x, y, z)$ at a point P decreases most rapidly in the direction of $v = i - 2j + 3k$. In this direction the value of the derivative is $-5\sqrt{14}$. The ∇f is:

a. $-5\sqrt{14}i + 10\sqrt{14}j - 15\sqrt{14}k$

b. $5\sqrt{14}i - 10\sqrt{14}j + 15\sqrt{14}k$

c. $-5i + 10j - 15k$

d. $5i - 10j + 15k$

e. None of the above

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5. The critical point(s) of $f(x, y) = e^{(x^2+y^2)}(2x^2 - y^2)$ is (are):

a. $(0, 0)$

b. $(0, 0), (\pm\frac{1}{\sqrt{2}}, 0)$

c. $(0, 0), (0, \pm\sqrt{2})$

d. $(0, 0), (\pm\frac{1}{\sqrt{2}}, 0), (0, \pm\sqrt{2})$

e. None of the above

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6. Let C be a simple closed curve, then $\int_C (x+y)dx + (x^2 - y^2)dy$ is the expression of:

a. The counterclockwise circulation of the field
$F = (x^2 - y^2)i + (x + y)j$ around C.

b. The counterclockwise circulation of the field
$F = (x^2 - y^2)i - (x + y)j$ around C.

c. The outward flux of the field
$F = (x + y)i + (x^2 - y^2)j$ across C

d. The outward flux of the field
$F = (x^2 - y^2)i - (x + y)j$ across C.

e. None of the above

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7. Use the chain rule to compute $\frac{\partial w}{\partial s}$ when $w = f(x, y, z)$, $x = 2s + t^2, y = tu$, and $z = u^2 - s^2$:

a. $\frac{\partial w}{\partial s} = (2s + t^2)\frac{\partial w}{\partial x} + tu\frac{\partial w}{\partial y} + (u^2 - s^2)\frac{\partial w}{\partial z}$

b. $\frac{\partial w}{\partial s} = 2\frac{\partial w}{\partial x} - 2s\frac{\partial w}{\partial z}$

c. $\frac{\partial w}{\partial s} = t\frac{\partial w}{\partial y} + 2u\frac{\partial w}{\partial z}$

d. $\frac{\partial w}{\partial s} = tu\frac{\partial w}{\partial y} + (u^2 - s^2)\frac{\partial w}{\partial z}$

e. None of the above

8. The value of the double integral $\int_0^2 \int_{y/2}^1 3ye^{x^3} dxdy$ is:

a. $2(e-1)$

b. $\frac{9}{2}(e-1)$

c. $\frac{25}{2}(e-1)$

d. $8(e-1)$

e. None of the above

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9. Let D be the region bounded by the paraboloids $z = 8-x^2-y^2$ and $z = x^2+y^2$. The volume of D in rectangular coordinates is expressed by the following iterated integral:

a. $\int_0^4 \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} dydxdz + \int_4^8 \int_{-\sqrt{8-z}}^{\sqrt{8-z}} \int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} dydxdz$

b. $\int_0^4 \int_{-\sqrt{8-z}}^{\sqrt{8-z}} \int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} dydxdz + \int_4^8 \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} dydxdz$

c. $\int_0^4 \int_{-\sqrt{8-z}}^{\sqrt{8-z}} \int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} dydxdz + \int_4^8 \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} dydxdz$

d. $\int_0^4 \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} dydxdz + \int_4^8 \int_{-\sqrt{8-z}}^{\sqrt{8-z}} \int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} dydxdz$

e. None of the above

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10. The value of the integral $I = \int_0^\infty e^{-x^2} dx$ is: (Hint: Compute $I^2 = (\int_0^\infty e^{-x^2} dx)(\int_0^\infty e^{-x^2} dx)$).

a. $\frac{\pi}{4}$

b. $\frac{\sqrt{\pi}}{2}$

c. $\sqrt{\frac{\pi}{2}}$

d. $\frac{\pi}{2}$

e. None of the above

Part II

I- (10 points) Use the transformation $u = x - y$ and $v = x + y$ to evaluate the double integral:

$$\int\int_R (x-y)^2 cos^2(x+y) dA$$

over the region R bounded by the lines: $x - y = 1$, $x - y = -1$, $x + y = 1$ and $x + y = 3$.

II- (10 points) Use the method of Lagrange Multipliers to find the dimensions of the rectangle of greatest area that can be inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

III- (10 points) Let D be the region bounded by below by the xy-plane, laterally by the cylinder $x^2 + y^2 = 9$, and above by the sphere $x^2 + y^2 + z^2 = 25$. Set up, but do not evaluate, the triple integral in **cylindrical** and **spherical** coordinates that represents the volume of D.

IV- (10 points) Consider the vector field

$$F = (y + z)i + (x - 2y)j + xk$$

a- Verify that the field is conservative.
b- Find a potential function $f(x, y, z)$ for F.
c- Use part b- to evaluate the work over C where C is a smooth curve whose inital point is (3,-2,1) and the terminal point is (-1,1,2)
d- Evaluate the work by finding parametrization for the segment that make up C.

V- (10 points) Let C be the closed curve in the plane starting from $(0, 0)$ which first goes to $(1, 1)$ along the parabola $y = x^2$ and then returns to $(0, 0)$ along the line $y = x$. Compute $\int_C y dx + e^y dy$ **twice**:

a) directly

b) using Green's theorem