

MAT 323-Vector & Tensor Analysis  
Exam # 2

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- 1) Find the work of the vector field  $\vec{F} = x\vec{i} + y\vec{j}$  along the quarter-circle parametrized by  $\vec{r}(t) = \cos t \vec{i} + \sin t \vec{j}$ ,  $0 \leq t \leq \pi/2$ . (15 points)
- 2) a) Sketch the plane triangle  $T$  in  $R^3$  parametrized by  $r(u, v) = (2u + v, v, 3u + v)$  for  $0 \leq u$ ,  $0 \leq v$ ,  $u + v \leq 1$ .  
b) Find the area of  $T$ . (20 points)
- 3) Let  $\Sigma$  be the helicoid parametrized by  $\vec{r}(u, v) = (u \cos v, u \sin v, v)$ ,  $0 \leq u \leq 1$ ,  $0 \leq v \leq \pi/2$ .

If  $\Sigma$  and its boundary  $C$  are oriented coherently, use Stokes' theorem to evaluate the work of  $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$  along  $C$ . (35 points)

- 4) Let  $S$  be the closed surface that is the boundary of the solid region inside the cylinder  $x^2 + y^2 = 4$  and between the planes  $z = 0$  and  $z = 2$ . Suppose  $S$  is positively oriented, with normal vector pointing out at each point. Find the flux of the vector field  $\vec{F} = (x^3, y^3 + x, xy)$  across  $S$ . (30 points)