

Vector & Tensor Analysis

Exam # 2

- 1) Let $\vec{F} = (x+y)\vec{i} + z\vec{j} + xz\vec{k}$. If Σ is the surface of the cube defined by $x = \pm 1$, $y = \pm 1$, $z = \pm 1$, what is flux of \vec{F} across Σ ?
(20 points)
- 2) Evaluate $\iint_{\Sigma} z \, ds$ where Σ is the portion of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 4$.
(25 points)
- 3) Let Q be the region bounded by the circular cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 3$. Let Σ denote the surface of Q . If $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^3\vec{k}$, use the divergence theorem to find the flux of \vec{F} across Σ .
(20 points)
- 4) Use Stokes' theorem to evaluate the work of the vector field $\vec{F} = (x+y)\vec{i} + (y+z)\vec{j} + (z+x)\vec{k}$ along the curve C which is the boundary of the triangle having vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$. Notice this triangle is obtained by intersecting the plane $x + y + z = 1$ with the first quadrant.
(Orient C counter clockwise)
(25 points)
- 5) Check if $\vec{F} = (3x^2 + y^2)\vec{i} + 2xy\vec{j} - 3z^2\vec{k}$ is conservative or not.
(10 points)