MAT 323-Vector & Tensor Analysis
Exam # 1

1) Evaluate the following.

   a) If $\vec{F} = (x^2 - y)\vec{i} + (xy - y^2)\vec{j}$, compute $Curl\vec{F}$.

   b) If $\vec{H} = \dfrac{x\vec{i} + y\vec{j} + z\vec{k}}{r^3}$ where $r = \sqrt{x^2 + y^2 + z^2}$, find $Div\vec{H}$.

(20 points)

2) Let P denote a point in rectangular cartesian coordinates $oxyz$.

   a) Find the angles $\alpha = (\overrightarrow{ox}, \overrightarrow{op})$, $\beta = (\overrightarrow{oy}, \overrightarrow{op})$, and $\gamma = (\overrightarrow{oz}, \overrightarrow{op})$.
   b) Let $ox'y'z'$ by the system obtained by rotating $oxyz$ about the z-axis through an angle α (in part a) from $\overrightarrow{ox}$ to $\overrightarrow{oy}$. Compute the rotation matrix T.
   c) Find the coordinates of the point p under $ox'y'z'$.

(20 points)

3) Let $f(x, y, z) = xe^y + yz$.
   a) What is the rate of change of f at the point $P_0 = (2, 0, 0)$ if a point $P(x, y, z)$ is moved straight toward $P_1 = (4, 1, -2)$.
   b) Find an equation for the tangent plane to the surface $xe^y + yz = 2$ at the point $(2, 0, 0)$.

(20 points)

4) Let $T = (l_{ij})$ be the transformation matrix of a rotation that takes $ox_1x_2x_3$ into $ox_1'x_2'x_3'$. If $P = (x_1, x_2, x_3)$ under $ox_1x_2x_3$ and $P = (x_1', x_2', x_3')$ under $ox_1'x_2'x_3'$, then

   a) Verify that $x_i' = l_{ij}x_j$ and $x_i' = l_{ik}x_k$.
   b) Show that $(x_i')^2 = (x_\alpha)^2$ without the use of geometry.

(20 points)

5) Let $\vec{F} = \left(x - \dfrac{xy}{x^2 + y^2} \,,\; y + \dfrac{x^2}{x^2 + y^2} \,,\; 0 \right)$.

   a) Express $\vec{F}$ in cylindrical frame.
   b) Find $div\vec{F}$ in terms of $r, \theta,$ and z.

(20 points)