

MAT 323-Vector & Tensor Analysis
Exam # 1

1) A set of axes $ox'y'z'$ is initially coincident with a set $oxyz$. The set $ox'y'z'$ is then rotated through an angle $\pi/4$ about the z -axis, the direction of rotation being from the x -axis to the y -axis.

- a- Find the transformation matrix of the rotation from $oxyz$ to $ox'y'z'$.
- b- If p has coordinates $(1,1,1)$ under $ox'y'z'$; what are the coordinates of p under $oxyz$?

2) Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and let $r = |\vec{r}|$.

a) Show that $\text{grad } r = \frac{\vec{r}}{r}$.

b- Show that $\text{grad } r^n = n r^{n-2} \vec{r}$.

c- Find a function whose gradient equals \vec{r} .

3) In spherical coordinates express the following vectors:

$$\vec{a} = \frac{\partial \vec{e}_\rho}{\partial \varphi}, \quad \vec{b} = \frac{\partial \vec{e}_\rho}{\partial \theta}, \quad \vec{c} = \frac{\partial \vec{e}_\theta}{\partial \theta}$$

in the form $F_\rho \vec{e}_\rho + F_\varphi \vec{e}_\varphi + F_\theta \vec{e}_\theta$.

4) Let $\vec{H} = -\frac{1}{r} \left(\frac{\partial f}{\partial z} \right) \vec{e}_r + \frac{1}{r} \frac{\partial f}{\partial r} \vec{e}_z$ denote a vector field expressed in cylindrical coordinates. If f is a function of r and z only, compute:

a- $\text{div } \vec{H}$

b- $\text{curl } \vec{H}$.