

MAT 323-Vector & Tensor Analysis
Exam # 1

- 1) a) Let $P_1 = (3, 4, 7)$ and $P_2 = (4, -1, 6)$ denote two points in the space. Find the direction cosines of the vector $\overline{P_1P_2}$.
b) Find the cosine of the angle between $\overline{P_1P_2}$ and the positive x -axis.
(10 points)
- 2) Given that $\vec{F} = x^2y\vec{i} + z\vec{j} - (x + y - z)\vec{k}$, find
a) $\text{div}\vec{F}$, $\text{curl}\vec{F}$, and $\overline{\text{grad}(\text{div}\vec{F})}$.
b) an expression for \vec{F} in cylindrical frame.
(30 points)
- 3) a right hand rectangular Cartesian coordinate system $ox_1x_2x_3$ is rotated about the x_3 -axis through an angle of $\frac{\pi}{4}$ to obtain the system $ox'_1x'_2x'_3$. The new system is then rotated about the x'_2 -axis through an angle of $\frac{\pi}{2}$ to obtain the system $ox''_1x''_2x''_3$.
a) Find the transformation matrices from $ox_1x_2x_3$ to $ox'_1x'_2x'_3$ and from $ox'_1x'_2x'_3$ to $ox''_1x''_2x''_3$.
b) Find the transformation matrix from $ox_1x_2x_3$ to $ox''_1x''_2x''_3$.
c) If A has components $(1, 1, 0)$ under $ox_1x_2x_3$, what are the components of A under $ox''_1x''_2x''_3$?
(25 points)
- 4) Let a coordinate system be defined by
$$u = x + y$$
$$v = 3y$$
$$w = y + z$$

a) Find the Cartesian components of \vec{e}_u , \vec{e}_v , and \vec{e}_w .
b) Is the system uvw orthogonal? Why?
(25 points)
- 5) The temperature of points in space is given by $f(x, y, z) = x^2 + y^2 - z$.
A mosquito located at $(1, 1, 2)$ desires to fly in such a direction that he will get cool as soon as possible. In what direction should he move?
(10 points)