

MAT 323-Vector & Tensor Analysis  
Exam # 1

1) Find the direction cosines of the line joining the origin to the point (6, 2, 5) (10 points)

2) Let  $ox'y'z'$  denote the new set of axes resulting from the rotation of  $oxyz$  through an angle  $\frac{\pi}{6}$  about the  $z$ -axis, the direction of rotation being from the  $x$ -axis to the  $y$ -axis.

- a- Find the transformation matrix of the above rotation.
  - b- If the point  $A$  has coordinates (2, 2, 2) under  $oxyz$ , what are the coordinates of  $A$  under  $ox'y'z'$ ?
- (20 points)

3) Consider the function  $f(x, y, z) = x^2 + y^2 + z - 9$ .

- a- Find the gradient of  $f$  at the point  $(x, y, z)$ .
  - b- Find equations for the tangent plane and normal of the surface  $f(x, y, z) = 0$  at the point (1, 2, 4).
- (20 points)

4) Let  $\vec{r} = x \vec{i} + y \vec{j} + z \vec{k}$  and  $r = \sqrt{x^2 + y^2 + z^2}$ .  
Evaluate the following

- a)  $\text{Curl} \left( \frac{\vec{r}}{r^2} \right)$
  - b)  $\text{Div} \left( \frac{\vec{r}}{r^2} \right)$
- (20 points)

5) a- Express the position vector  $\vec{r} = x \vec{i} + y \vec{j} + z \vec{k}$  in cylindrical coordinates frame.

- b- Find  $\frac{\partial \vec{r}}{\partial \theta}$  in cylindrical frame.
- (20 points)

6) Use orthonormality conditions to show that the quantity  $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 = \overline{x_i x_i}$  is invariant under rotation. (10 points)