FINAL EXAM.; MATH 201

January 28, 2004

Name: Signature:

Student number:

Section number (Encircle):

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Instructor: Prof. Abdallah Lyzzaik

1. Instructions:

- Calculators are allowed.

- There are two types of questions: **PART I** consists of **six subjective** questions, and **PART II** consists of **seven multiple-choice** questions of which each has exactly one correct answer.

- GIVE DETAILED SOLUTIONS FOR THE PROBLEMS OF **PART I** IN THE PROVIDED SPACE AND CIRCLE THE APPROPRIATE ANSWER FOR EACH PROBLEM OF **PART II**.

2. Grading policy:

- 12 points for each problem of **PART I**.

- 4 points for each problem of **PART II**: 0 point for no answer, -1 for a wrong answer or more than one answer of **PART II**.

GRADE OF PART I/72:

GRADE OF PART II/28:

TOTAL GRADE/100:

Part I (1). Find the absolute maximum and minimum values attained by the function $f(x, y) = xy - x - y + 3$ on the triangular region R in the $xy-$plane with vertices $(0, 0)$, $(2, 0)$, and $(0, 4)$.

Part I (2). Evaluate the integral

$$\int_0^2 \int_{y/2}^1 y\, e^{x^3}\, dx\, dy.$$

Part I (3). Set up a triple integral (without evaluating it) in cylindrical coordinates for the volume of the solid bounded by the $xy-$plane, the cylinder $r^2 = \cos 2\theta$, and the sphere $x^2 + y^2 + z^2 = 1$.

Part I (4). Evaluate the integral

$$\int\int_R \sin\left(\frac{y-x}{y+x}\right)\, dx\, dy,$$

where R is the trapezoid in the $xy-$plane with vertices $(1,1)$, $(2,2)$, $(4,0)$, and $(2,0)$, by making the change of variables: $u = y - x$, $v = y + x$.

Part I (5). Evaluate the line integral

$$\oint_C 3xy\,dx + 2x^2\,dy,$$

where C is the boundary of the region R bounded above by the line $y = x$ and below by the parabola $y = x^2 - 2x$. Interpret this integral in terms of vector fields.

Part I (6). Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(2x-1)^n}{\ln n}.$$

State where the series converges absolutely and conditionally.

Part II

1. If $f(x,y) = 2x^2y/(x^4+y^2)$, then

   (a) $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

   (b) $\lim_{(x,y)\to(0,0)} f(x,y) = 1$.

   (d) $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

   (c) $\lim_{(x,y)\to(0,0)} f(x,y) = 2$.

   (e) None of the above.

2. An estimate of the integral

$$\int_0^1 \frac{1-\cos x}{x^2}\,dx$$

with an error less than $1/(6!5)$ is

   (a) $1/2! + 1/(4!3)$.

   (b) $1/2! - 1/(4!3)$.

   (c) $1/2! - 1/(4!3) + 1/(6!5)$.

   (d) $-1/2! + 1/(4!3) - 1/(6!5)$.

   (e) None of the above.

3. The function defined by

$$f(x,y) = \tan\left(\frac{x^3-y^3}{x^2+y^2}\right)$$

for $(x,y) \neq (0,0)$, and $f(0,0) = 0$

   (a) has no limit at $(0,0)$.

   (b) has a limit at $(0,0)$ but is discontinuous at $(0,0)$.

   (c) is continuous at $(0,0)$.

   (d) is bounded in the $xy-$plane.

   (e) None of the above.

4. If $w = f(x,y)$ where $x = e^r \cos\theta$ and $y = e^r \sin\theta$, then

   (a) $w_{xx} + w_{yy} = -w_{rr} + w_r/r + w_{\theta\theta}/r^2$.

   (b) $w_{xx} + w_{yy} = w_{rr} + w_r/r - w_{\theta\theta}/r^2$.

   (c) $w_{xx} + w_{yy} = w_{rr} - w_r/r + w_{\theta\theta}/r^2$.

   (d) $w_{xx} + w_{yy} = w_{rr} + w_r/r + w_{\theta\theta}/r^2$.

   (e) None of the above.

5. An equation of the tangent plane to the surface with equation $z^3 + xz - y^2 = 1$ at the point $(1, 3, 2)$ is

   (a) $2x - 6y + 13z = 10$.

   (b) $2x + 6y + 13z = 10$.

   (c) $2x + 6y - 13z = 10$.

   (d) $2x - 6y + 13z = -10$.

   (e) None of the above.

6. The volume of the solid bounded by the cylinder $y = x^2$ and the planes $y + z = 4$ and $z = 0$ is given by the triple integral

   (a) $\int_0^4 \int_0^{4-y} \int_{-\sqrt{y}}^{\sqrt{y}} dx\, dy\, dz$.

   (b) $\int_0^4 \int_0^{4-z} \int_{-\sqrt{y}}^{\sqrt{y}} dx\, dy\, dz$.

   (c) $2 \int_0^4 \int_{\sqrt{y}}^2 \int_0^{4-y} dz\, dx\, dy$.

   (d) $\int_{-2}^2 \int_0^{x^2} \int_0^{4-y} dz\, dy\, dx$.

   (e) None of the above.

7. The function $f(x, y) = x^3 + 3xy - y^3$ admits

   (a) a saddle point and no local minimum value.

   (b) a local minimum value and no saddle point.

   (c) a saddle point and a local minimum value.

   (d) no saddle point and no local minimum value.

   (e) None of the above.