NOTRE DAME UNIVERSITY

MAT 113

EXAM 2

DATE: WEDNESDAY JANUARY 9, 2008
DURATION: 55 MINUTES

NAME: --

ID NUMBER: --

SECTION: --

Key

GRADE: --

1-(6 points) Find the function of derivative $g'(x) = 4x^3 + 3$ and whose graph passes through the point P (2, 23)

Sol. $g(x) = x^4 + 3x + C$ (4) pts replace P(2, 23)

$\Rightarrow 23 = 16 + 6 + C \Rightarrow C = 23 - 22 = 1$

$\therefore g(x) = x^4 + 3x + 1.$

(2) pts

2-(10 Points) Suppose that $\int_{-3}^{0} g(t)dt = \sqrt{2}$, find:

a) $\int_{0}^{-3} [-g(t)]dt = -\int_{0}^{-3} g(t)\,dt = \int_{-3}^{0} g(t)\,dt = \sqrt{2}.$ (5) pts

b) $\int_{-3}^{0} \frac{g(t)}{\sqrt{2}} dt = \frac{1}{\sqrt{2}} \int_{-3}^{0} g(t)\,dt = \frac{1}{\sqrt{2}}(\sqrt{2}) = 1.$ (5) pts

3-(14 Points) Find the value or values of c that satisfy the equation $\frac{f(b)-f(a)}{b-a} = f'(c)$

for the function $f(x) = x + \frac{1}{x}$ over $\left[\frac{1}{2}, 2\right]$

Sol.

4 pts

* $f(x) = x + \frac{1}{x}$ cont on $[\frac{1}{2}, 2]$.
* $f'(x) = 1 - \frac{1}{x^2}$ exists on $(\frac{1}{2}, 2)$

$\therefore$ we can find at least one value $c \in (\frac{1}{2}, 2)$, so that

3 pts $\frac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}} = f'(c) \Rightarrow \frac{\frac{5}{2} - \frac{5}{2}}{3/2} = f'(c) \Rightarrow f'(c) = 0$

$f(2) = 2 + \frac{1}{2} = \frac{5}{2}$

$f(\frac{1}{2}) = \frac{1}{2} + 2 = \frac{5}{2}$

$\Rightarrow 1 - \frac{1}{c^2} = 0$

$\Rightarrow \frac{c^2 - 1}{c^2} = 0 \Rightarrow c^2 = 1 \Rightarrow c = \pm 1$

7 pts

$\therefore \boxed{c = 1} \in (\frac{1}{2}, 2)$.

4-(15 Points) Find the average value of $f(x) = \sqrt{16 - x^2}$ on $[-4, 0]$.
(Hint: you may use areas to evaluate definite integrals)

5 pts

Sol.

$\text{ave } f = \frac{1}{0-(-4)} \int_{-4}^{0} \sqrt{16 - x^2}\, dx = \frac{1}{4} \int_{-4}^{0} \sqrt{16 - x^2}\, dx$.

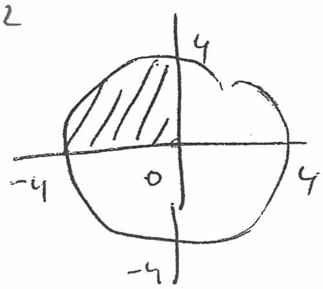
Consider: $y = \sqrt{16 - x^2}$ on $[-4, 0]$

$y^2 = 16 - x^2$

$x^2 + y^2 = 16$

Center $(0, 0)$

$R = 4$



3 pts

$= \frac{1}{4}\left(\frac{\text{area of a circle}}{4}\right)$

$= \frac{1}{4}\left(\frac{\pi r^2}{4}\right)$

$= \frac{1}{16} \pi (4)^2$

$= \frac{1}{16}(16\pi) = \pi$.

7 pts

5-(15 Points) Find the local extreme values of the function $f(x)=\dfrac{x}{x^2+1}$ and where they occur.

Sol: (2 pts) Dom f: $\mathbb{R}=(-\infty,\infty)$. $\Rightarrow$ expected positions for local extremes at critical pts only.

(6 pts)

$$f'(x)=\frac{x^2+1-2x(x)}{(x^2+1)^2}=\frac{x^2+1-2x^2}{(x^2+1)^2}=\frac{1-x^2}{(x^2+1)^2}.$$

$$f'(x)=0 \Rightarrow 1-x^2=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1.$$

$\therefore$ 2 critical pts $\left(-1, f(-1)=-\frac{1}{2}\right)$

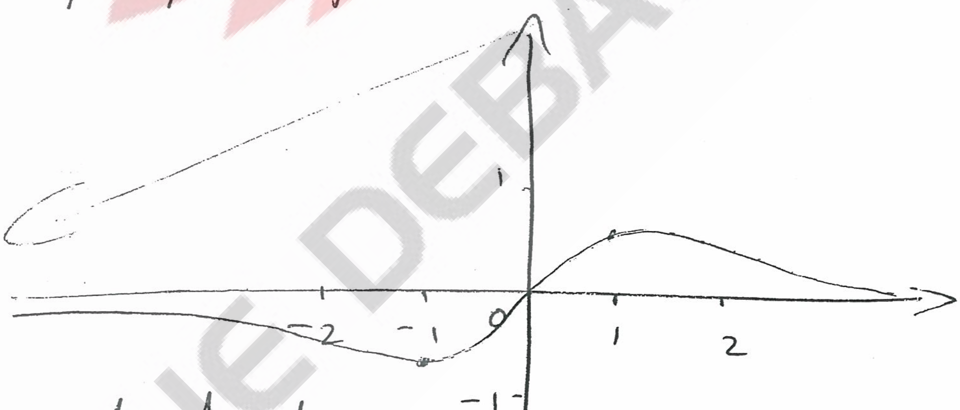
$\left(1, f(1)=\frac{1}{2}\right)$.

$-1 \quad 1$

$- \quad + \quad -$

(7 pts)

x	$-\infty$	-2	-1	0	1	2	∞
f(x)	0^-	-0.4	-0.5	0	0.5	0.4	0^+



$\therefore$ at $x=-1$, local and Abs min $=-0.5$.

at $x=1$, local and Abs max $=0.5$.

6-(15 Points) Find the absolute extreme values of the function $g(x) = x^3 - 3x + 4$, $-2 \le x \le 0$.

Sol: g(x) is cont on a closed interval $[-2,0] \Rightarrow$ Abs extremes exists and expected to be at a critical pt or endpt. (3 pts)

* Critical pts $\Rightarrow g'(x)=0$ or $g'(x)$ undefined?

$g'(x) = 3x^2 - 3 = 3(x^2-1) = 0 \Rightarrow x = \pm 1 \Rightarrow x=-1 \in [-2,0]$ (4 pts)

$\therefore$ only critical pt in $[-2,0]$ is $(-1, g(-1)=6) = (-1,\underline{6})$

* End pts: $(-2, g(-2) = 2) = (-2, \underline{2})$ (1 pt)

$(0, 4)$

$\therefore$ Abs max $= 6$ at $x=-1$ (1 pt)

Abs min $= 2$ at $x=-2$. (1 pt)

7-(8 Points) Find $\frac{dy}{dx}$, *Given that* $y = \int_{x^2}^{3} \frac{2}{t+3}\, dt$.

Sol: $y = \int_{x^2}^{3} \frac{2}{t+3}\, dt = -\int_{3}^{x^2} \frac{2}{t+3}\, dt$ (3 pts)

$y' = -\left(\frac{2}{x^2+3}\right)(2x) = \frac{-4x}{x^2+3}$.

$y' = f'(u) \cdot u'(x)$, $u = x^2$. (5 pts)

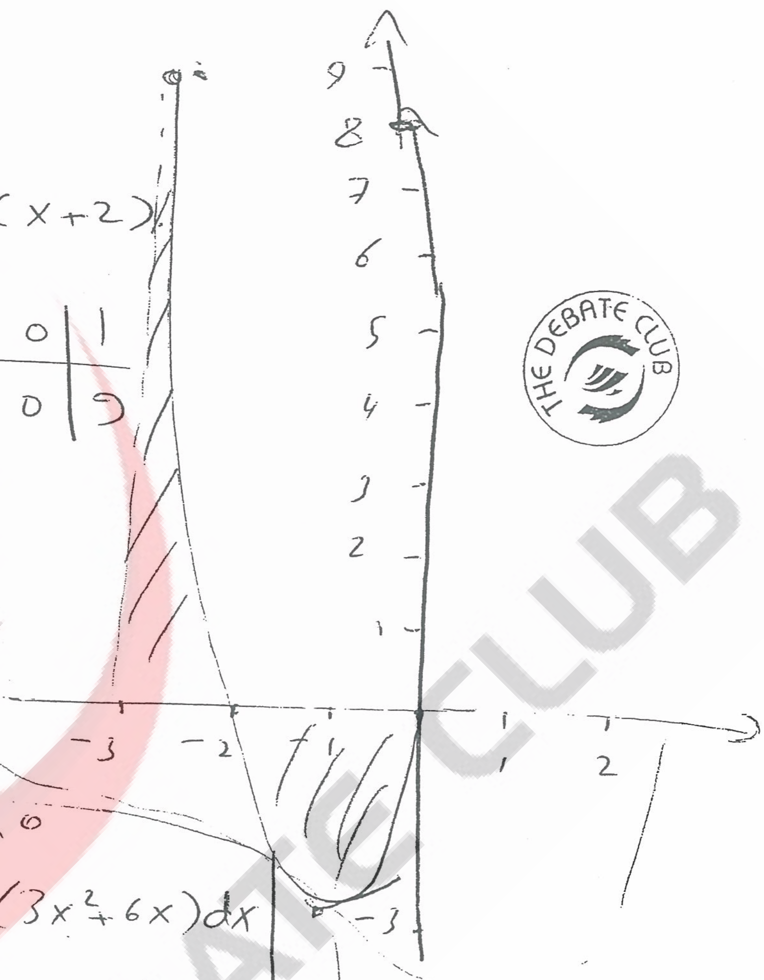
8-(17 Points) Find the area of the region bounded between the curve of $f(x)=3x^2+6x$ and the X-axis over the interval $[-3,0]$

Sol.

$f(x) = 3x^2+6x = 3x(x+2)$

x	-4	-3	-2	-1	0	1
$f(x)$		9	0	-3	0	9

(3) pts



$$A = \int_{-3}^{-2}(3x^2+6x)\,dx + \left|\int_{-2}^{0}(3x^2+6x)\,dx\right|$$

(5) pts

$$= x^3+3x^2\Big]_{-3}^{-2} + \left|x^3+3x^2\Big]_{-2}^{0}\right|$$

(5) pts

$$= \left[-8+12-(\cancel{-27+27})\right] + \left|0-[-8+12]\right|$$

$$= 4 + \left|-[4]\right| = 4 + |-4| = 4+4 = 8\,u^2.$$

(4) pts