Final Exam.; Math. 201; Jan. 31, 1996; 11:00 A.M.-1:00 P.M.

Name: Signature:

Student number:

Section number (Encircle): 3 5 6

1. Instructions:

   - No calculators are allowed.
   - Make sure that you have exactly 20 questions.
   - Each question has only one correct answer.
   - Make sure to circle your answers appropiately below.

2. Grading policy:

   - 5 points for each correct answer.
   - -1 point (penalty) for each wrong answer.
   - 0 point for zero or more than one answer.

Circle the appropiate answers:

1.	a	b	c	d	11.	a	b	c	d
2.	a	b	c	d	12.	a	b	c	d
3.	a	b	c	d	13.	a	b	c	d
4.	a	b	c	d	14.	a	b	c	d
5.	a	b	c	d	15.	a	b	c	d
6.	a	b	c	d	16.	a	b	c	d
7.	a	b	c	d	17.	a	b	c	d
8.	a	b	c	d	18.	a	b	c	d
9.	a	b	c	d	19.	a	b	c	d
10.	a	b	c	d	20.	a	b	c	d

(0)

1. The area of the region lying inside the circle $r = -3\cos\theta$ and outside the cardioid $r = 1 - \cos\theta$ is

(a) $3(\pi + 3\sqrt{2} - 1)$.

(b) π.

(c) 2π.

(d) $2(\pi + 2\sqrt{3} - 2)$.

2. The points of intersection of the curves $r = 2$ and $r = 4\cos 2\theta$ are

(a) $(2,\ 0)$, $(2,\ \pi)$, $(2,\ \pm\pi/2)$.

(b) $(2,\ \pm\pi/6)$, $\pm\pi/3$, $(2,\ \pm2\pi/3)$, $(2,\ \pm5\pi/6)$.

(c) $(2,\ \pm\pi/12)$, $\pm5\pi/12$, $(2,\ \pm13\pi/12)$, $(2,\ \pm17\pi/12)$.

(d) $(2,\ \pm\pi/4)$, $(2,\ \pm5\pi/4)$.

3. The surface $x^2/16 - y^2/25 = z/4$ is

(a) a circular paraboloid.

(b) a hyperbolic paraboloid.

(c) an elliptic cone.

(d) a one-sheeted hyperboloid.

4. If $f(x,y) = x^2y^3/[x^8 + y^4]$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$, then

(a) $\lim_{(x,y)\to(0,0)} f(x,y) = 1/2$.

(b) f is discontinuous at (0,0).

(c) $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

(d) f is continuous at (0,0).

(1)

5. An estimate of the error obtained by taking the first four terms of the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}(0.01)^n/n^3 \quad \text{is}$$

(a) 1.56×10^{-10}.

(b) 8×10^{-11}.

(c) 1.56×10^{-12}.

(d) 8×10^{-13}.

6. The interval of convergence of the power series

$$\sum_{n=0}^{\infty} 2^n(x-3)^n/n^2 \quad \text{is}$$

(a) $5/2 < x < 7/2$.

(b) $1 \le x \le 5$.

(c) $5/2 \le x \le 7/2$.

(d) $1 < x < 5$.

7. The Maclaurian series of the integral

$$\int_0^x \frac{sint^4}{t^2}dt \quad \text{is}$$

(a) $\sum_{n=0}^{\infty}(-1)^n x^{8n+1}/(8n+1)[(2n+1)!]$.

(b) $\sum_{n=0}^{\infty}(-1)^n x^{8n+3}/(8n+3)[(2n+1)!]$.

(c) $\sum_{n=0}^{\infty}(-1)^n x^{8n+1}/(8n+3)[(2n+1)!]$.

(d) $\sum_{n=0}^{\infty}(-1)^n x^{8n+3}/(8n+3)[(2n+3)!]$.

(2)

8. If $a_n = (\frac{3}{5})^n + \frac{4^n}{n!}$ and $b_n = n(e^{\frac{1}{n}} - 1)$, then

(a) the sequences $\{a_n\}$ and $\{b_n\}$ diverge.

(b) the sequences $\{a_n\}$ and $\{b_n\}$ converge.

(c) the sequence $\{a_n\}$ diverges and $\{b_n\}$ converges.

(d) the sequence $\{a_n\}$ converges and $\{b_n\}$ diverges.

9. If $a_n = \frac{2n+1}{n^2(n+1)^2}$ and $b_n = \frac{1}{n(\ln n)^{ln 2}}$, then

(a) the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge.

(b) the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge.

(c) the series $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ converges.

(d) the series $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges.

10. If $a_n = (1 - \frac{1}{n})^n$ and $b_n = \frac{\cos n\pi}{\sqrt{n}}$, then

(a) the series $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ converges.

(b) the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge.

(c) the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge.

(d) the series $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges.

11. The Taylor series of $\ln x$ at $x = 3$ is

(a) $\ln 3 + \sum_{n=1}^{\infty} (-1)^n (x-3)^n / n3^n$.

(b) $\ln 3 + \sum_{n=1}^{\infty} (-1)^{n+1} (x-3)^n / n!3^n$.

(c) $\ln 3 + \sum_{n=1}^{\infty} (-1)^{n+1} (x-3)^n / n3^n$.

(d) $\ln 3 + \sum_{n=1}^{\infty} (-1)^n (x-3)^n / n!3^n$.

(3)

12. If $z = f(x, y)$ where $x = r\cos\theta$ and $x = rsin\theta$, then

(a) $f_x^2 + f_y^2 = r^2 f_r^2 + f_\theta^2$.

(b) $f_x^2 + f_y^2 = f_r^2 + r^{-2} f_\theta^2$.

(c) $f_x^2 + f_y^2 = f_r^2 + f_\theta^2$.

(d) $f_x^2 + f_y^2 = f_r^2 - f_\theta^2$.

13. An equation of the tangent plane to the surface $x^2+2xy+2y^2+z = 12$ at the point $P(2, -3, 2)$is

(a) $-8x + 18y - z = -72$.

(b) $2x + 8y - z = -22$.

(c) $2x + 18y - z = -52$.

(d) $-8x + 8y - z = -42$.

14. The tangent line to the curve of intersection of the paraboloid $x^2 + y^2 - z = 0$ and the ellipsoid $2x^2+3y^2+z^2-9 = 0$ through the point $P(1, -1, 2)$ has parametric equations:

(a) $x = 1 + 7t, y = -1 + 6t, z = 2 + 2t$.

(b) $x = 1 + 5t, y = -1 - 7t, z = 2 + t$.

(c) $x = 1 - 3t, y = -1 + 9t, z = 2 + 3t$.

(d) $x = 1 + 4t, y = -1 - 7t, z = 2 - t$.

(4)

15. The rate of change of the temperature function $T(x, y) = 4x^2 - y^2 + 16z^2$ at the point $P(-1, 2, 1)$ is zero in the direction of the vector:

(a) $< 3, 4, -2 >$.

(b) $< 1, -1, 2 >$.

(c) $< 1, -2, 0 >$.

(d) $< 0, 8, -1 >$.

16. The function $f(x, y) = 3x^2 - 4xy + 5y^2 - 7$ admits -7 as a

(a) local maximum value.

(b) an absolute minimum value.

(c) a value at a saddle point.

(d) an absolute maximum value.

17. The value of the double integral

$$\int_0^2 \int_{y/2}^1 y e^{x^3} \, dx dy \quad \text{is}$$

(a) $(e - 1)/4$.

(b) $5(e - 1)/4$.

(c) $2(e - 1)/3$.

(d) $(e - 1)/3$.

(5)

18. The average value of the function $e^{-(x^2+y^2)}$ over the first quadrant of the disc $x^2 + y^2 \leq 1$ is

(a) $(e-1)/2$.

(b) $e-1$.

(c) $1-e^{-1}$.

(d) $(1-e^{-1})/2$.

19. The volume of the solid bounded by the paraboloids $z = 2x^2 + y^2$ and $z = 12 - x^2 - 2y^2$ is

(a) 20π.

(b) 24π.

(c) 18π.

(d) 22π.

20. The solid whose volume is given by the triple integral in spherical coordinates as

$$\int_0^{2\pi} \int_0^{\pi/6} \int_0^{2cos\phi} \rho^2 sin\phi d\rho d\phi d\theta$$

is bounded by

(a) a cone and a cylinder.

(b) a sphere and a cylinder.

(c) a cone and a sphere.

(d) a sphere and a paraboloid.

(6)