

**Tuesday April 9, 2002**

1) (18 points) Evaluate the following integrals.

a)  $\int \sin \frac{x}{2} \cos^3 \frac{x}{2} dx$

b)  $\int_0^2 \sqrt{4-x^2} dx$

c)  $\int \frac{(1-\sqrt{x})^4}{\sqrt{x}} dx$

2) (10 points) Find the mean value of the function  $f(x) = |x^2 - 1|$  over the interval  $[0, 2]$ .

3) (10 points) If  $\int_0^x f(t) dt = \sin x + \tan x$ . Find  $f\left(\frac{\pi}{4}\right)$ .

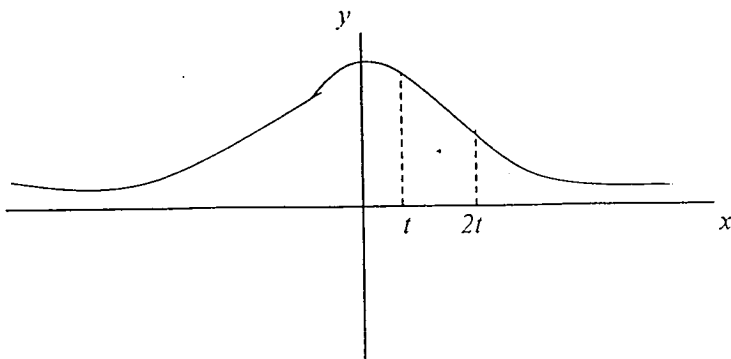
4) (16 points) Find the area of the region between the graph of  $f(x) = x^2 - 4$  and the  $x$ -axis from  $x = 0$  to  $x = 3$ .

5) (26 points) Find the volume of the solid generated by revolving the region enclosed by the curve  $y = \sqrt{2x}$ , the line  $x = 8$  and the line  $y = 2$ , around

a) The line  $y = 2$ .

b) The  $y$ -axis.

6) (20 points) Below is the graph of the function  $f(x) = \frac{1}{x^2 + 1}$ .



a) If  $t$  is a real positive variable and  $y(t)$  is defined as  $y(t) = \int_t^{2t} f(x) dx$ , find  $\frac{dy}{dt}$ .

b)  $y(t)$  (Defined in part (a)) represents the area of the region between the graph of  $f(x)$  and  $x$ -axis from  $t$  to  $2t$ . Find  $t$  that maximizes this area.