FINAL EXAM.; MATH 201

February 6, 1998; 8:00-10:00 A.M.

Name: Signature:

Student number:

Section number (Encircle): 3 10 11 12

Instructors (Encircle): Prof. H. Abu-Khuzam Prof. A. Lyzzaik

1. Instructions:

- No calculators are allowed.
- There are two types of questions: **PART I** consisting of four subjective questions, and **PART II** consisting of twelve multiple-choice questions of which each has exactly one correct answer.
- GIVE DETAILED SOLUTIONS FOR THE PROBLEMS OF **PART I** IN THE PROVIDED SPACE AND CIRCLE THE APPROPIATE ANSWERS FOR THE PROBLEMS OF **PART II**.

2. Grading policy:

- 10 points for each problem of **PART I**.
- 5 points for each problem of **PART II**.
- 0 point for no answer, wrong answer, or more than one answer of **PART II**.

GRADE OF PART I/40:

GRADE OF PART II/60:

TOTAL GRADE/100:

Part I(1). Find the absolute maximum and minimum values of the function $f(x, y) = x^3 + 3xy - y^3$ on the triangular region R with vertices $(1, 2)$, $(1, -2)$, and $(-1, -2)$.

Part I(2). Evaluate the integral

$$\int_0^1 \int_x^{\sqrt{x}} e^{x/y}\,dy\,dx.$$

Part I(3). Set up a triple integral (without evaluating it)in cylindrical coordinates for the volume of the solid bounded by the $xy-$plane, the cylinder $r = 1 + \sin\theta$, and the plane $x + y + z = 2$.

Part I(4). Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{1}{n5^n}(x-5)^n.$$

State where the series converges absolutely and conditionally.

Part II

1. The area of the region lying outside the circle $r = 3$ and inside the cardioid $r = 2(1 + cos\theta)$ is

   (a) $\frac{9}{2}\sqrt{3} - \pi$.

   (b) $\frac{9}{2}\sqrt{3} + \pi$.

   (c) $9\sqrt{3} + \pi/2$.

   (d) $9\sqrt{3} - \pi/2$.

   (e) None of the above.

2. The slope of the tangent line to the curve $r = 8\cos 3\theta$ at the point of the graph corresponding to $\theta = \pi/4$ is

   (a) 2.

   (b) -2.

   (c) 0.

   (d) 1.

   (e) None of the above.

3. If $f(x, y) = \frac{x^3y^2}{x^4+y^8}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$, then

   (a) $\lim_{(x,y)\to(0,0)} f(x, y) = 1/2$.

   (b) f is discontinuous at (0,0).

   (c) $\lim_{(x,y)\to(0,0)} f(x, y) = 0$.

   (d) $\lim_{(x,y)\to(1,-1)} f(x, y) = 2$.

   (e) None of the above.

4. An estimate to four decimal places of the value of the integral

$$\int_0^{0.1} x^2 e^{-x^2} dx \quad \text{is}$$

(a) 10^{-4}.

(b) 2×10^{-4}.

(c) 5×10^{-4}.

(d) 3×10^{-4}.

(e) None of the above.

5. The Maclaurin series of the integral

$$\int_0^{x} \sqrt[3]{1+t^2}\, dt \quad \text{is}$$

(a) $\sum_{n=1}^{\infty} \frac{(\frac{1}{3})(\frac{1}{3}-1)\cdots(\frac{1}{3}-n+1)}{n!(2n+1)} x^{2n+1}$.

(b) $x + \sum_{n=1}^{\infty} \frac{(\frac{1}{3})(\frac{1}{3}-1)\cdots(\frac{1}{3}-n+1)}{n!(2n+1)} x^{2n+1}$.

(c) $x - \sum_{n=1}^{\infty} \frac{(\frac{1}{3})(\frac{1}{3}-1)\cdots(\frac{1}{3}-n+1)}{n!(2n+1)} x^{2n+1}$.

(d) $x + \sum_{n=1}^{\infty} \frac{(\frac{1}{3})(\frac{1}{3}-1)\cdots(\frac{1}{3}-n+1)}{(2n+1)} x^{2n+1}$.

(e) None of the above.

6. If $a_n = (\frac{7}{2})^n + \frac{e^n}{n!}$ and $b_n = n^2(e^{1/n^2} - 1)$, then

(a) the sequences $\{a_n\}$ and $\{b_n\}$ diverge.

(b) the sequences $\{a_n\}$ and $\{b_n\}$ converge.

(c) the sequence $\{a_n\}$ diverges and $\{b_n\}$ converges.

(d) the sequence $\{a_n\}$ converges and $\{b_n\}$ diverges.

(e) None of the above.

7. The sum of the series

$$\sum_{n=0}^{\infty}[(-1)^n \frac{(\pi/2)^{2n+1}}{(2n+1)!} + \frac{n}{3^{n-1}}] \quad \text{is}$$

(a) 15/4.

(b) 5/4

(c) 7/4

(d) 13/4

(e) None of the above.

8. The function defined by $f(x, y) = \cos\left(\frac{x^3-y^3}{x^2+y^2}\right)$ for $(x, y) \neq (0, 0)$, and $f(0, 0) = 1$

(a) has no limit at $(0, 0)$.

(b) has a limit at $(0, 0)$ but is not continuous at $(0, 0)$.

(c) is continuous at $(0, 0)$.

(d) is unbounded.

(e) None of the above.

9. If $z = f(x, y)$ where $x = e^r cos\theta$ and $y = e^r sin\theta$, then

(a) $f_x^2 - f_y^2 = e^{-2r}(f_r^2 - f_\theta^2)$.

(b) $f_x^2 + f_y^2 = e^{-2r}(f_r^2 + f_\theta^2)$.

(c) $f_x^2 + f_y^2 = e^{2r}(f_r^2 - f_\theta^2)$.

(d) $f_x^2 + f_y^2 = e^{2r}(f_r^2 + f_\theta^2)$.

(e) None of the above.

10. An equation of the tangent plane to the ellipsoid $\frac{3}{4}x^2 + 3y^2 + z^2 = 12$ at the point $P(2, 1, \sqrt{6})$ is

(a) $3x - 6y + 2\sqrt{6}z = 12$.

(b) $3y - 6x + 2\sqrt{6}z = 3$.

(c) $3y + 6x + 2\sqrt{6}z = 27$.

(d) $3x + 6y + 2\sqrt{6}z = 24$.

(e) None of the above.

11. If the directional derivatives of $f(x, y)$ at the point $P(1, 2)$ in the direction of the vector $\mathbf{i} + \mathbf{j}$ is $2\sqrt{2}$ and in the direction of the vector $-2\mathbf{j}$ is -3, then f increases most rapidly at P in the direction of the vector

(a) $3\mathbf{i} - \mathbf{j}$.

(b) $3\mathbf{i} + \mathbf{j}$.

(c) $\mathbf{i} - 3\mathbf{j}$.

(d) $\mathbf{i} + 3\mathbf{j}$.

(e) None of the above.

12. The function $f(x, y) = \frac{1}{3}x^3 + \frac{4}{3}y^3 - x^2 - 3x - 4y - 3$ admits

(a) a local maximum value $4/3$.

(b) a local minimum value $-42/3$.

(c) a saddle point $(3, 1, f(3, 1))$.

(d) an absolute maximum value $f(1, 1)$.

(e) None of the above.