

Mat 324-Mathematics for Engineering
Final exam



1. Use Green's theorem to evaluate the work of the force $\vec{F} = (x^5 + 3y)\vec{i} + (2x - e^{y^3})\vec{j}$ along the circle $(x-1)^2 + (y-5)^2 = 4$.
(10 pts)

2. The electric field due to a point charge q at the origin is given by the force $\vec{E} = kq \vec{r} / \|\vec{r}\|^3$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $\|\vec{r}\|$ = length of \vec{r} , and k is a constant.
Determine the flux of \vec{E} out of the sphere $x^2 + y^2 + z^2 = a^2$
(20 pts)

3.
 - a) Locate the 4 roots of $z^4 = 1 - i$ on the appropriate circles
 - b) Write $f(z) = \text{Im}(z - 3\bar{z}) + z \text{Re}(z^2) - 5z$ in the form $f(z) = u(x, y) + iv(x, y)$
 - c) If $e^z = 2i$, then find z .

(15pts)

4.
 - a) If $f(z) = \int_C \frac{s^2 + 6s - 2}{s - z} ds$, where C is $|z| = 3$, then what is the value of $f(1 + i)$?
 - b) Draw two simple curves C_1 and C_2 in the right half plane $\text{Re}(z) > 0$ between $z = 1 + i$ and $z = 3 + 2i$. Then show that $\int_{C_1} \frac{z-2}{z} dz = \int_{C_2} \frac{z-2}{z} dz$.
 - c) Let C be the circle $|z| = 1$. Evaluate $\int_C \frac{dz}{z}$ and $\int_C z^n dz$ for $n \in \mathbb{Z}$.

(15pts)

5.
 - a) Determine the residue of $f(z) = (z-i)^2 \sin \frac{1}{z-i}$ at $z = i$ by using the Laurent series of $f(z)$.
 - b) Determine the order of the zero, $z = 0$, of the function $f(z) = z(e^z - 1)$
 - c) Evaluate $\int_C \frac{e^{2z}}{z^4 + 2z^3 + 2z^2} dz$, where C is the circle $|z| = 1$

(15pts)

6. Evaluate the integral

$$\int_0^{\infty} \frac{2x^2 - 1}{x^4 + 5x^2 + 4} dx$$

(25pts)