Mat 324 – Mathematics for Engineering
Exam # 2

Solve the following problems

1) a – Let $f(z) = 3x - y + 5 + i(ax + by - 3)$.
   Determine a and b in order for f to be analytic.

   b – If $f(z) = x^2 + y^2 + (2xy)i$, find when the Cauchy Riemann Equations are satisfied and show that f is not analytic at any point

(20pts)

2) a - Express $\mathrm{Log}(\sqrt{3} + i)$ in the a + ib form

   b - Is it true that $\mathrm{Log}\, i^3 = 3\mathrm{Log}\, i$? Why ?

   c - If $f(z) = z^{24} - 3z^{20} + 4z^{12} - 5z^6$ and $z_0 = \dfrac{(1+i)}{\sqrt{2}}$, evaluate $f(z_0)$

   (Hint:Express z_0 in exponential form)

(20pts)

3) If f is a polynomial and C is a simple closed contour, then what is the value of $\oint_C f(z)\,dz$?

(15pts)

4) If $f(z) = z^3 + e^z$ and C is the contour $z = 8e^{it}$, $0 \le t \le 2\pi$.

   find $\displaystyle\int_C \frac{f(z)}{(z + \pi i)^3}\,dz$

(15pts)

5) If $|f(z)| \le 2$ on the circle C: $|z| = 3$, find an upper bound for $\left|\int_C f(z)\,dz\right|$

(15pts)

6) Expand the function $f(z) = \dfrac{\sin z}{z^4}$ in a Laurent series valid for $|z| \ge 0$

(15pts)

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64

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THE DEBATE CLUB

N° 1

a) f is analytic
f has derivative
Let's check the cauchy-rieman equation
$f(z) = 3x - y + 5 + i(ax + by - 3)$
$U_x = 3$ $V_y = b$ $\Rightarrow U_x = V_y \Rightarrow b = 3$
$V_x = a$ $U_y = -1$ $V_x = -U_y \Rightarrow a = 1$

$f(z) = 3x - y + 5 + i(3x + y - 3)$

b) $f(z) = x^2 + y^2 + (2xy)i$
$U_x = 2x$ $V_y = 2x$
$U_y = 2y$ $V_x = 2y$

14

Cauchy rieman equation is satisfied
when $f(z) = (x+iy)^2 = x^2 - y^2 + i(2xy)$
In this case f is analytic
In $f(z) = x^2 + y^2 + (2xy)i$
Cauchy rieman equation is not satisfie
so f has no derivative
f is not analytic at any point

N° 2

a) $\text{Log}(\sqrt{3} + i)$
$= \text{Log } z$?
$e^z = re^{i\theta}$
$r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$e^x \cdot e^{iy} = re^{i\theta}$

$x \Rightarrow \ln 2$

$y = \pi/6$

$\mathrm{Log}(\sqrt{3}+i) = \ln 2 + i(\pi/6)$ ✓

b- $\mathrm{Log}\, i^3 \overset{?}{=} 3\,\mathrm{Log}\, i$

$e^z = re^{i\theta}$

$e^x \cdot e^{iy} = e^0 e^{i\pi/2} = e^{i\pi/2}$

$\mathrm{Log}\, i^3 = \ln 1 + i(\pi/2 + 2k\pi) = i(\pi/2 + 2k\pi)$

$3\,\mathrm{Log}\, i \overset{?}{=}$ $\mathrm{Log}\, i = \ln 1 + i(\pi/2 + 2k\pi) \Rightarrow 3\,\mathrm{Log}\, i = 3i(\pi/2 + 2k\pi)$

$\boxed{\mathrm{Log}\, i^3 = -\frac{\pi}{2} i,\ 3\,\mathrm{Log}\, i = 3\frac{\pi}{2} i.}$

12

c- $f(z) = z^{24} - 3z^{20} + 4z^{12} - 5z^6$

$z_0 = \frac{(1+i)}{\sqrt{2}}$

$r = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{2}} = 1$

$z_0 = e^{i\pi/4}$

$f(z_0) = \left(e^{i\pi/4}\right)^{24} - 3\left(e^{i\pi/4}\right)^{20} + 4\left(e^{i\pi/4}\right)^{12} - 5\left(e^{i\pi/4}\right)^6$

$= ?$

N° 3

$\int_c f(z)\,dz = 0$ because $f(z)$ is analytic everywhere

why is f analytic everywhere and if this so why the integral is zero.

8

N° 4

$$f(z) = z^3 + e^z$$
$$= (8e^{it})^3 + e^{8e^{it}}$$

$$dz = 8ie^{it}\,dt$$

$$\int_0^{2\pi} \frac{\left((8e^{it})^3 + e^{8it}\right)}{(8e^{it} + \pi i)^3}(8ie^{it})\,dt$$

Zero

N° 5

$$|f(z)| \le 2 = M$$
$$L = 2\pi\rho = 6\pi$$

$$\left|\int_c f(z)\,dz\right| = M.L = 2 \times 6\pi = 12\pi$$

15

N° 6

$$f(z) = \frac{\sin z}{z^4}$$

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} - - - + \frac{z^n}{n!} + \cdots$$

$$\frac{\sin z}{z^4} = \frac{1}{z^3} - \frac{1}{6z} + \frac{z}{5!} - \frac{z^3}{7!} + \cdots + \frac{z^{n-4}}{n!} + \cdots$$

$$f(z) = \frac{1}{z^3} - \frac{1}{6z} + \frac{z}{5!} - \frac{z^3}{7!} + \cdots + \frac{z^{n-4}}{n!} + \cdots$$

15