MAT 324 – Mathematics for Engineering
Exam#1

Solve the following problems.

1) Evaluate $\int_C y\,dx + x\,dy$ where C is the portion of the curve $y = x^2$ stretching from $(0,0)$ to $(1,1)$.

(15 points)

2) Find a function $\varphi(x,y)$ so that $\overrightarrow{grad}\,\varphi = 2xy\,\vec{i} + x^2\,\vec{j}$. Then evaluate $\int_{(1,1)}^{(2,4)} 2xy\,dx + x^2\,dy$.

(20 points)

3) Determine the work of the force $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ along the curve C which is boundary of the surface of the plane $2x + y + 2z = 6$ lying in the first octant. (Specify the orientation of C).

(20 points)

4) a) Sketch in the xyz-coordinates the region D described by $1 \le x \le 3$, $0 \le y \le 3$, and $0 \le z \le \frac{1}{x}$.

b) Find the volume of the region D.

c) Use the divergence theorem to find the flux of the force $\vec{F} = \vec{i} + \frac{1}{3Ln3} y\vec{j} + \vec{k}$ across the boundary of the region D.

(25 points)

5) Find all solutions of the equation $z^8 - 2z^4 + 1 = 0$. Locate the solutions on the appropriate circle.

(20 points)

N° 1

$I = \int y\,dx + x\,dy$ $\quad C: y = x^2 \quad (0,0)\ (1,1)$

parametrize C: $\quad x = t \quad y = t^2 \quad 0 \leq t \leq 1$

$$I = \int_0^1 t^2 dt + t(2t)\,dt = \int_0^1 3t^2\,dt = t^3\Big|_0^1 = 1$$

N° 2

Find φ so that grad $\varphi = 2xy\,\vec{i} + x^2\vec{j}$

$$\text{grad}\,\vec{\varphi} = \frac{d\varphi}{dx}\vec{i} + \frac{d\varphi}{dy}\vec{j}$$

Set $\dfrac{d\varphi}{dx} = 2xy \qquad \dfrac{d\varphi}{dy} = x^2$

$$\frac{d\varphi}{dy} = x^2 \Rightarrow \varphi = \int x^2 dy = x^2 y + g(x)$$

$$\varphi_x = 2xy + g'(x) = 2xy \Rightarrow g'(x) = 0$$

$$g(x) = C$$

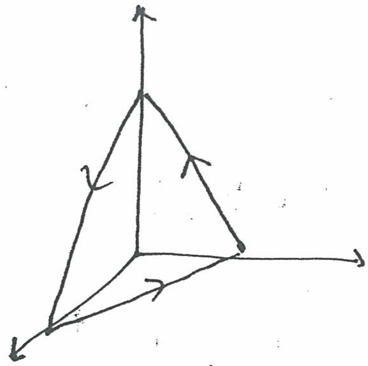
Hence $\varphi(x,y) = x^2 y + C$

$$\int_{(1,1)}^{(2,4)} (2xy)\,dx + x^2 dy = \int_{(1,1)}^{(2,4)} d\varphi = \varphi(2,4) - \varphi(1,1)$$

$$= 16 - 1$$

$$= 15$$

N°3



By def

Work of $\vec{F} = \int_C \vec{F} \cdot d\vec{r}$

But we can say
work of $\vec{F}$ = flux of curl $\vec{F}$ across the surface whose boundary is C

step1: $\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z & x & y \end{vmatrix} = \vec{i} + \vec{j} + \vec{k}$

Flux of curl $\vec{F} = \iint_S \text{curl}\,\vec{F} \cdot \vec{n}\, ds$

step 2: finding $\vec{n}$

the plane is obtained by setting $g(x,y,z) = 2x + y + 2z = 6$

$\text{grad}\,\vec{g} = 2\vec{i} + \vec{j} + 2\vec{k}$

$|\text{grad}\,\vec{g}| = 3$

$\vec{n} = \frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}$

$\text{curl}\,\vec{F} \cdot \vec{n} = 2/3 + 1/3 + 2/3 = 5/3$

work $= \iint_S 5/3\, ds = 5/3$ Area of S

Area of S $= \frac{1}{2} |\vec{AB} \times \vec{AC}|$

Finding Area of S (using integral)

S is described by

$z = \frac{1}{2}(6 - 2x - y) = f(x,y)$

Area $= \iint_R \sqrt{f_x^2 + f_y^2 + 1}\, dA$

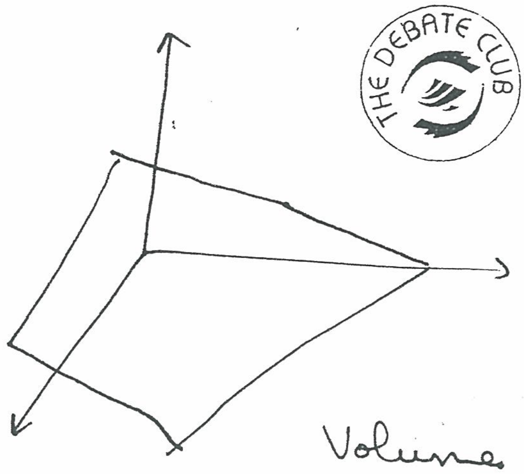
$= \iint_R \sqrt{1 + 1/4 + 1}\, dA = \iint 3/2\, dA = 3/2$ area of R

Area of $R = \frac{6\times 3}{2} = 9$

Conclusion : Surface Area $= \frac{3}{2}\times 9 = 27/2$

Work $= 5/3 \times \frac{27}{2} = 45/2$

N° 4



Volume $= \iiint dx\,dy\,dz$

$$= \int_1^3\int_0^3\int_0^{1/x} dz\,dy\,dx = \int_1^3\int_0^3 z\Big|_0^{1/x} dy\,dx$$

$$= \int_1^3 \frac{1}{x} y\Big|_3^0 dx$$

$$= 3\int_1^3 \frac{1}{x}\,dx$$

$$= 3\ln x\Big|_1^3 = 3\ln 3$$

Flux of $\vec{F}$ across $S = \iiint_D \operatorname{div}\vec{F}\,dV$

$\operatorname{div}\vec{F} = 1/3 \ln 3$

Flux $= \iiint \frac{1}{3}\ln 3\,dV = 1/3 \ln 3$ volume of D

$$= \frac{3\ln 3}{3\ln 3} = 1$$

N° 5

$$z^8 - 2z^4 + 1 = 0$$

$$(z^4)^2 - 2z^4 + 1 = 0$$

$$(z^4 - 1)^2 = 0$$

$$z^4 = 1$$

$$z = re^{i\theta}$$

$$r^4 e^{i4\theta} = e^{i(0 + 2k\pi)}$$

$$r^4 = 1 \Rightarrow r = 1$$

$$4\theta = 2k\pi$$

$$\theta = \frac{k\pi}{2}$$

$k = 0 \Rightarrow \theta = 0$ $\quad z_0 = 1$

$k = 1 \Rightarrow \theta = \pi/2$ $\quad z_1 = e^{i\pi/2} = i$

$k = 2 \Rightarrow \theta = \pi$ $\quad z_2 = e^{i\pi} = -1$

$k = 3 \Rightarrow \theta = 3\pi/2$ $\quad z_3 = e^{i3\pi/2} = -i$