FINAL EXAMINATION (A)

MATH 201

January 22, 2007; 8:00-10:00 P.M.

Name: Signature:

Student number:

Section number:

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Instructions:

- No calculators are allowed.
- There are two types of questions:

PART I consists of six work-out problems. Give a detailed solution.

PART II consists of eight multiple-choice questions each with **exactly one correct answer**. Circle the appropriate answer.

Grading policy:

- 10 points for each problem of **PART I**.
- 5 points for each problem of **PART II**.
- 0 point for no, wrong, or more than one answer of **PART II**.

GRADE OF PART I	/60
GRADE OF PART II	/40
TOTAL GRADE	/100

Part I (1). Use Lagrange Multipliers to find the point on the plane $2x + 3y + 4z = 12$ at which $f(x, y, z) = 4x^2 + y^2 + 5z^2$ has its least value.

Part I (2). Sketch the solid bounded by the cone $z = \sqrt{x^2 + y^2}$, the cylinder $x^2 + y^2 = 4$, and below by the $xy-$plane, and use triple integrals to find its volume.

Part I (3). Find the absolute maximum and minimum values of the function $f(x, y) = 5 + 4x - 2x^2 + 3y - y^2$ over the triangular region R bounded by the lines $y = 2$, $y = x$ and $y = -x$.

Part I (4). Evaluate the integral

$$\iint_R \frac{e^{x-y}}{x+y}\, dA,$$

where R is the rectangle bounded by the lines $y = x$, $y = x + 5$, $y = 2 - x$ and $y = 4 - x$ by using the substitution $x - y = u$ and $x + y = v$.

Part I (5). Use Green's theorem to evaluate the line integral

$$\oint_C (7y - e^{\sin x})dx + [15x - \sin(y^3 + 8y)]dy,$$

where C is the positively-directed cardioid $r = 1 + \cos\theta$.

Part I (6). Sketch the region of integration and evaluate the integral

$$\int_0^1 \int_y^1 e^{x^2}\, dx\, dy.$$

Part II (1). The value of the double integral

$$\int\int_R \cos(x^2 + y^2)\ \ dx\ dy,$$

where R is the region bounded by the circle $x^2 + y^2 = 9$ is

(a) $2\pi \sin 3$.

(b) $\pi \sin 3$.

(c) $2\pi \sin 9$.

(d) $\pi \sin 9$.

(e) None of the above.

Part II (2). The mass of the solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$ and whose density function is $\delta(x, y, z) = (x^2 + y^2)^{3/2}$ is given by the triple integral

(a) $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{3/2}\ dz\ dx\ dy$.

(b) $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{3/2}\ dz\ dy\ dx$.

(c) $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{3/2}\ dz\ dy\ dx$.

(d) $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{2-x^2-y^2}^{x^2+y^2} (x^2 + y^2)^{3/2}\ dz\ dy\ dx$.

(e) None of the above.

Part II (3). The volume of the solid bounded above by the sphere $x^2+y^2+z^2=4$, below by the $xy-$plane, and lies inside the cylinder $x^2+y^2=1$ is given by the triple integral

(a) $\int_0^{2\pi}\int_0^{\pi/3}\int_0^2 \ \rho^2\sin\phi\ d\rho\ d\phi\ d\theta+\int_0^{2\pi}\int_{\pi/3}^{\pi/2}\int_0^{1/\cos\phi}\ \rho^2\sin\phi\ d\rho\ d\phi\ d\theta.$

(b) $\int_0^{2\pi}\int_0^{\pi/6}\int_0^2 \ \rho\sin\phi\ d\rho\ d\phi\ d\theta+\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{1/\sin\phi}\ \rho\sin\phi\ d\rho\ d\phi\ d\theta.$

(c) $\int_0^{2\pi}\int_0^{\pi/6}\int_0^2 \ \rho^2\sin\phi\ d\rho\ d\phi\ d\theta+\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{\sin\phi}\ \rho^2\sin\phi\ d\rho\ d\phi\ d\theta.$

(d) $\int_0^{2\pi}\int_0^{\pi/6}\int_0^2 \ \rho^2\sin\phi\ d\rho\ d\phi\ d\theta+\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{1/\sin\phi}\ \rho^2\sin\phi\ d\rho\ d\phi\ d\theta.$

(e) None of the above.

Part II (4). The approximate value of the integral

$$\int_0^{0.1}\frac{\sin x}{x}\ dx$$

with an error of magnitude less than 10^{-4} is

(a) 0.1.

(b) 0.2.

(c) 0.3.

(d) 0.4.

(e) None of the above.

Part II (5). The power series

$$\sum_{k=2}^{\infty} (-1)^k \frac{(x-1)^k}{k \ln k}$$

has interval of convergence

(a) $[0, 2]$.

(b) $[0, 2[$.

(c) $]0, 2]$.

(d) $]0, 2[$.

(e) $]-\infty, \infty[$.

Part II (6). The direction of maximum change for the function

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$

at the point $(1, 2, -2)$ is

(a) $\mathbf{u} = (2/3)\mathbf{i} + (2/3)\mathbf{j} + (-1/3)\mathbf{k}$.

(b) $\mathbf{u} = (1/3)\mathbf{i} + (-2/3)\mathbf{j} + (2/3)\mathbf{k}$.

(c) $\mathbf{u} = (1/3)\mathbf{i} + (2/3)\mathbf{j} + (-2/3)\mathbf{k}$.

(d) $\mathbf{u} = (-1/3)\mathbf{i} + (2/3)\mathbf{j} + (2/3)\mathbf{k}$.

(e) $\mathbf{u} = (-1/3)\mathbf{i} + (-2/3)\mathbf{j} + (-2/3)\mathbf{k}$.

Part II (7) The function $f(x, y) = 2y^2 - x^3 - 2xy$

(a) has local minimum at $(-1/3, -1/6)$ and local maximum at $(0, 0)$.

(b) has local minimum at $(-1/3, -1/6)$ and saddle point at $(0, 0)$.

(c) has local maximum at $(-1/3, -1/6)$ and saddle point at $(0, 0)$.

(d) has local maximum at $(-1/3, -1/6)$ and local minimum at $(0, 0)$.

(e) has saddle point at $(-1/3, -1/6)$ and local minimum at $(0, 0)$.

Part II (8) Equations of the tangent plane and normal line to the surface $z = 6 - x^2 - y^2$ at the point $(1, 2, 1)$ are respectively

(a) $2x + 4y + z = 11$ and $(x - 1)/2 = (y - 2)/4 = (z - 1)$.

(b) $x + 4y + 2z = 11$ and $(x - 1) = (y - 2)/4 = (z - 1)/2$.

(c) $4x + 2y + 3z = 11$ and $(x + 1)/2 = (3y - 2)/4 = (z + 1)/2$.

(d) $3x + 2y + 4z = 11$ and $(2x - 1)/2 = (y - 1)/2 = (2z - 1)/2$.

(e) $4x + 3y + z = 11$ and $(4x - 1)/3 = (y + 2)/4 = (4z - 1)/3$.