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Find the Taylor polynomial of degree 5 at x = 0 for the function f (x) = x² sin 2 x

$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} - - - - = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

$\sin 2x \approx 2x - \frac{(2x)^3}{6} + \frac{(2x)^5}{5!} - - - - - = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!}$

$x^2 \sin 2x \approx 2x^3 - \frac{8x^5}{6} + \frac{32x^7}{5!} - - - -$

Taylor polynomial of degree 5 at $x=0$ for the function $f(x) = x^2 \sin 2x$

is $2x^3 - \frac{8x^5}{6}$.

Answer $x^2 \sin 2x = 2x^3 - \frac{8x^5}{6}$ ✓

For what values of x can we replace cos x by $1 - \frac{x^2}{2} + \frac{x^4}{24}$ with an error $\leq 3 \times 10^{-5}$?

$$\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

$$|\text{error}| \leq \frac{|x|^6}{6!} \leq 3\times 10^{-5}$$

$$|x|^6 \leq 3\times 10^{-5} \times 6!$$

$$|x|^6 \leq 0.0216$$

$$|x| \leq \sqrt[6]{0.0216}$$

$$\boxed{|x| \leq 0.5277}$$

Answer $|x| \leq 0.5277$

Given $f(x) = \ln\left(\frac{1+x}{1-x}\right) = \sum_{n=1}^{\infty} \frac{2}{2n-1} x^{2n-1}$ $\quad -1 < x < 1$

Use a Taylor polynomial for f (x) of degree 5 to find an approximate value for ln 2

$\ln\left(\frac{1+x}{1-x}\right) \approx 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5$

ln 2 ??

$\frac{1+x}{1-x} = 2$

$1+x = 2-2x$

$3x = 1$

$\Rightarrow \boxed{x = \frac{1}{3}}$ ✓

Replace x by $1/3$

$\Rightarrow \ln 2 \approx \ln\left(\frac{1+1/3}{1-1/3}\right) = 2\left(\frac{1}{3}\right) + \frac{2}{3}\left(\frac{1}{3}\right)^3 + \left(\frac{2}{5}\right)\left(\frac{1}{3}\right)^5$

$= \frac{2}{3} + \frac{2}{81} + \frac{2}{1215}$

$= \frac{810+30+2}{1215} = \frac{842}{1215}$ ✓

$\boxed{\ln 2 \approx 0.693}$

Answer	$\ln 2 \approx 0.693$

Use series to evaluate $\lim_{x \to 0} \dfrac{\sin x - x + \frac{x^3}{6}}{x^5}$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} - - - - = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cancel{\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}}$$

$$\sin x - x + \frac{x^3}{6} \approx \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - - -$$

$$\frac{\sin x - x + \frac{x^3}{6}}{x^5} \approx \frac{1}{5!} - \frac{x^2}{7!} + \frac{x^4}{9!} - - - \xrightarrow[x \to 0]{} \frac{1}{5!}$$

$$\Rightarrow \boxed{\lim_{x \to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5} = \frac{1}{120} = \frac{1}{5!}}$$

Answer $\lim_{x \to 0} \dfrac{\sin x - x + \frac{x^3}{6}}{x^5} = \dfrac{1}{120}$

Find the radius and interval of convergence

of the series $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$ $\quad a = 5/4$

$$\frac{(4x-5)^{2n+3}}{(4x-5)^{2n+1}} \cdot \frac{n^{3/2}}{(n+1)^{3/2}} = |4x-5|^2 \cdot \frac{n^{3/2}}{(n+1)^{3/2}} \xrightarrow[n\to\infty]{} |4x-5|^2 =$$

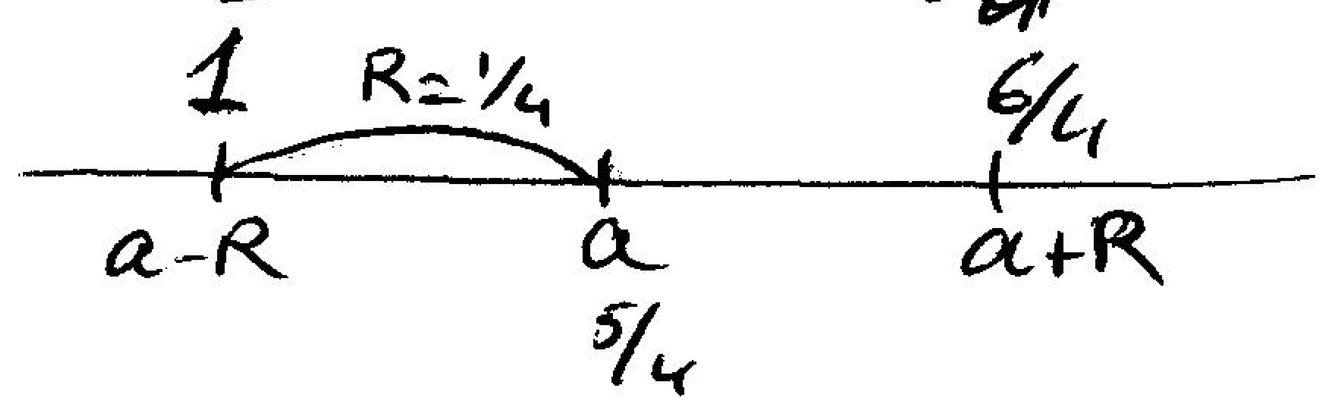
the series converges for $\rho < 1$ by the Ratio test.

$$\Longrightarrow \quad |4x-5|^2 < 1$$

$$-1 < 4x - 5 < 1$$

$$4 < 4x < 6$$

$$\boxed{1 < x < 6/4} \longrightarrow \text{Radius} = 1/4$$



we check:

$x = 1$: $\longrightarrow \sum_{n=1}^{\infty} \frac{(-1)^{2n+1}}{n^{3/2}}$ alt. p-series $p > 0$ $\Longrightarrow$ converges

$x = 6/4$: $\longrightarrow \sum_{n=1}^{\infty} \frac{(1)^{2n+1}}{n^{3/2}}$ converges p series, $p > 1$

$\Longrightarrow$ interval of convergence is $\boxed{1 \leq x \leq 6/4}$

Answer	**Radius of Convergence**	$R = 1/4$
	Interval of Convergence	$1 \leq x \leq 6/4 \; 3/2$