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Find the Taylor polynomial of degree 4
at $x = 0$ for the function $f(x) = x^2 \cos 2x$

$$\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

Taylor polynomial of degree 4
for the function $\cos x$

we replace x by $2x$

$$\cos 2x \approx 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{24}$$

Taylor polynomial for the function
 $\cos 2x$

$$x^2 \cos 2x \approx x^2 - 2x^4 \quad \checkmark$$

Taylor polynomial for the function
 $f(x) = x^2 \cos 2x$ at $x=0$ of degree 4

For what values of x can we replace $\cos x$

by $1 - \frac{x^2}{2} + \frac{x^4}{24}$ with an error $\leq 3 \times 10^{-5}$?

$$\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

$$|\text{error}| \leq \frac{|x|^6}{6!}$$

$$\frac{|x|^6}{6!} \leq 3 \times 10^{-5}$$

$$|x|^6 \leq (3 \times 10^{-5}) \cdot 6!$$

$$|x| \leq \sqrt[6]{(3 \times 10^{-5}) \cdot 6!}$$

$$|x| \leq \sqrt[6]{216 \cdot 10^{-4}} \approx 0.527$$

$$|x| \leq 0.527$$