Haigazian University **24 May 2010**
MATH 202 Test 3 **Solution**

[1]

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$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+6}\right\}$$
$$=\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+2}\right\}=\frac{1}{\sqrt{2}}\,e^{-2t}\sin\left(\sqrt{2}\,t\right)$$

[2]
Find the **Laplace transform** of the function

$$f(t)=\begin{cases}0 & 0\le t<1\\ t^2 & t\ge 1\end{cases}$$

$t^2 = (t-1)^2 + 2(t-1) + 1$

$$f(t)=t^2\,\mathcal{U}(t-1)$$
$$=(t-1)^2\,\mathcal{U}(t-1)+2(t-1)\,\mathcal{U}(t-1)+\mathcal{U}(t-1)$$

??

$$\mathcal{L}\{f(t)\}=\underbrace{\frac{2}{s^3}e^{-s}+\frac{2}{s^2}e^{-s}}+\frac{1}{s}e^{-s}=\frac{s^2+2s+2}{s^3}e^{-s}$$

[3]
Use the Laplace transform to solve the **boundary value problem**

$$y''+2y'+y=0$$
$$y'(0)=2$$
$$y(1)=2$$

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$$s^2Y(s)-s\,y(0)-2+2(s\,Y(s)-y(0))+Y(s)=0$$
$$(s+1)^2Y(s)=(s+1)\,y(0)+2+y(0)$$
$$Y(s)=y(0)\frac{1}{s+1}+(2+y(0))\frac{1}{(s+1)^2}$$
$$y(t)=y(0)\,e^{-t}+(2+y(0))\,t\,e^{-t}$$
$$2=e^{-1}(2\,y(0)+2)$$
$$y(0)=e-1$$
$$y(t)=(e-1+(e+1)\,t)\,e^{-t}$$

[4]

$-F'(s)$

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$$\int_0^\infty e^{-st}\,t\cos t\,dt=\mathcal{L}\{t\cos t\}=\cancel{\frac{s^2-1}{(s^2+1)^2}}$$
$$\int_0^\infty e^{-2t}\,t\cos t\,dt=\frac{3}{25}$$