Haigazian University 26 Apr 2010
MATH 202 Test 2

1. Use **variation of parameters** to find a particular solution y_p of the differential equation $2x^2y''+5xy'+y=3\sqrt{x}$, $x>0$

Solution:

The auxiliary equation $2m^2+3m+1=0$ has two roots $m_1=-\frac{1}{2}$ and $m_2=-1$

Therefore $y_p=u_1x^{-1/2}+u_2x^{-1}$

$W=-\frac{1}{2}x^{-5/2}$ $\quad W_1=-\frac{3}{2}x^{-5/2}$ $\quad W_2=\frac{3}{2}x^{-2}$

$u_1'=3$ $\quad u_2'=-3x^{1/2}$ $\quad u_1=3x$ $\quad u_2=-2x^{3/2}$ $\quad y_p=\sqrt{x}$

2. Find solutions in the form of **power series** in x for the differential equation

$$y''-(x+1)\,y'-y=0$$

Solution:

$$\sum_{n=2}^{\infty} n\,(n-1)\,c_n x^{n-2} - (x+1)\sum_{n=1}^{\infty} n\,c_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)\,(n+1)\,c_{n+2}x^n - \sum_{n=0}^{\infty} n\,c_n x^n - \sum_{n=0}^{\infty} (n+1)\,c_{n+1}x^n - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} \{(n+2)\,(n+1)\,c_{n+2} - n\,c_n - (n+1)\,c_{n+1} - c_n\}\,x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)\,\{(n+2)\,c_{n+2} - c_{n+1} - c_n\}\,x^n = 0$$

Recurrence relation:

$$c_{n+2} = \frac{c_{n+1}+c_n}{n+2} \qquad n \ge 0$$

$$c_0\left(1+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{6}+\frac{x^5}{15}+\ldots\right)+c_1\left(x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^4}{4}+\frac{3x^5}{20}+\ldots\right)$$

3. Find two linearly independent solutions y_1 and y_2 of the differential equation

$$x^2y''+(3-k)xy'-2ky=0 \qquad (x>0)$$

where y is a function of x and k is a real number

Solution:

$$m^2+(2-k)m-2k=(m-k)(m+2)=0$$

Fundamental set of solutions:

$\{x^k, x^{-2}\}$ if $k\neq -2$

$\{x^{-2}, x^{-2}\ln x\}$ if $k=-2$