

1. Use the method of **Undetermined Coefficients** to solve the initial value problem.

$$y'' + 4y' = 16x$$

$$y(0) = 0$$

$$y'(0) = 4$$

Answer: $y = \frac{5}{4} - \frac{5}{4}e^{-4x} + 2x^2 - x$

2. Find the general solution of the differential equation $x^2 y'' - 3x y' + 4y = 0$ on the interval $(0, \infty)$ given that $y_1 = x^2$ is a solution.

Answer: $y = c_1 x^2 + c_2 x^2 \ln x$

3. Determine a homogeneous linear differential equation with constant coefficients having the solutions: $3, x e^{-2x}, -e^{-2x}$

Answer: $\frac{d^3 y}{dx^3} + 4 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} = 0$

4. Given the family of curves

$$y = \frac{cx}{1+x}$$

find the member of the **orthogonal trajectories** that passes through $(1, 1)$

Answer: $3y^2 + 3x^2 + 2x^3 = 8$