Haigazian University

MATH 202 Quiz 7

Solution

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Consider the homogeneous second order **Cauchy–Euler** differential equation

$$\textbf{(H)} \qquad x^2 y'' + kxy' - ky = 0 \qquad\qquad (x > 0)$$

where y is a function of x and k is a real number.

1) Show that $y = x$ is a solution of **(H)**

$$y' = 1 \quad y'' = 0$$
$$x^2 y'' + kxy' - ky = x^2\,0 + kx\,1 - kx = kx - kx = 0$$

2) Find two linearly independent solutions y_1 and y_2 of **(H)**

The auxiliary equation:

$$m^2 + (k-1)m - k = 0$$
$$(m-1)(m+k) = 0$$

The roots $\quad m_1 = 1 \quad m_2 = -k$

If $k \neq -1 \quad y_1 = x \quad y_2 = x^{-k}$

If $k = -1 \quad y_1 = x \quad y_2 = x \ln x$

3) Compute the **Wronskian** determinant to check that y_1 and y_2 are linearly independent on the interval $x > 0$

If $k \neq -1$ $\quad W = \begin{vmatrix} x & x^{-k} \\ 1 & -kx^{-k-1} \end{vmatrix} = -kx^{-k} - x^{-k} = -(k+1)x^{-k} \neq 0$

If $k = -1$ $\quad W = \begin{vmatrix} x & x\ln x \\ 1 & 1+\ln x \end{vmatrix} = x \neq 0$