**Haigazian University** **12 Apr 2010**
MATH 202 Quiz 6

Use **variation of parameters** to solve the differential equation

$$y''+2y'+y=\frac{e^{-x}}{x}$$

subject to the **boundary** conditions: $y(1)=0 \quad y(2)=0$

**Solution:**

First, solve the corresponding homogeneous differential equation

$$y''+2y'+y=0$$

The auxiliary equation $m^2+2m+1=0$ has a repeated root $m_1=m_2=-1$

Therefore $y_c=c_1e^{-x}+c_2xe^{-x}$.

Next, find a particular solution of the nonhomogeneous equation of the form

$$y_p=u_1e^{-x}+u_2xe^{-x}$$

where
$$u_1'=\frac{\begin{vmatrix} 0 & xe^{-x} \\ e^{-x}/x & e^{-x}-xe^{-x} \end{vmatrix}}{\begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & e^{-x}-xe^{-x} \end{vmatrix}}=\frac{-e^{-2x}}{e^{-2x}}=-1 \quad , \quad u_1=-x$$

and
$$u_2'=\frac{\begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & e^{-x}/x \end{vmatrix}}{\begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & e^{-x}-xe^{-x} \end{vmatrix}}=\frac{e^{-2x}/x}{e^{-2x}}=\frac{1}{x} \quad , \quad u_2=\ln x$$

0 **General Solution:**

$$y=c_1e^{-x}+c_2xe^{-x}+xe^{-x}\ln x=e^{-x}(c_1+c_2x+x\ln x)$$

0 **Boundary Conditions**

$$0=c_1+c_2$$
$$0=c_1+2c_2+2\ln 2$$

**Answer:** $y=e^{-x}(2\ln 2-2x\ln 2+x\ln x)$