Haigazian University **22 Mar 2010**

MATH 202 Quiz 5A

Find a second solution of the differential equation $x^2y'' - 20y = 0$ given $y_1 = x^{-4}$

Solution: Use the formula $y_2(x) = y_1 \int \frac{e^{-\int p(x)dx}}{y_1^2} dx$ with $p(x) = 0$

$$y_2(x) = x^{-4} \int \frac{1}{x^{-8}} dx = x^{-4} \int x^8 \, dx = x^{-4} \frac{x^9}{9} = \frac{x^5}{9}$$

Solve the initial value problem

$$\begin{cases} y'' - 8y' + 17y = 0 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$$

Solution:

The characteristic equation $m^2 - 8m + 17 = 0$ has two complex roots

$4 + i$ and $4 - i$ so that the general solution is given by

$$y = e^{4x}(c_1 \cos x + c_2 \sin x)$$

Differentiate to get

$$y' = e^{4x}(4c_1 \cos x + c_2 \cos x + 4c_2 \sin x - c_1 \sin x)$$

Using the initial conditions we get

$$c_1 = 1$$

$$c_2 = --3$$

Answer: $y = e^{4x}(\cos x - 3\sin x)$