Haigazian University **15 Mar 2010**
MATH 202 Quiz 4B

Find the member of the orthogonal trajectories for $y(2x^3 + c) = 6$ that passes through the point $(1, 1)$

Solution:

Differentiate the given equation and then eliminate c to obtain the differential equation satisfied by the given family of curves

$$y'(2x^3 + c) + y\,6x^2 = 0$$

$$y'\,\frac{6}{y} + y\,6x^2 = 0$$

$$y' + x^2y^2 = 0$$

Next, replace y' with $\dfrac{-1}{y'}$ to get the differential equation for the orthogonal trajectories

$$\frac{-1}{y'} + x^2y^2 = 0$$

$$x^2y^2y' = 1$$

It is a separable differential equation

$$y^2\,dy = \frac{1}{x^2}\,dx$$

Integrate $\quad \dfrac{1}{3}y^3 = \dfrac{-1}{x} + c_2$

Replace $x = 1$ $y = 1$ to find the value of $c_2 = \dfrac{4}{3}$

Answer: $y^3 + \dfrac{3}{x} = 4$