

Solve the initial value problem

$$y' + xy = x, \quad y(0) = 6$$

An integrating factor

$$v(x) = e^{\int x dx} = e^{\frac{1}{2} x^2}$$

$$e^{\frac{1}{2} x^2} y' + x e^{\frac{1}{2} x^2} y = x e^{\frac{1}{2} x^2}$$

$$\frac{d}{dx} (e^{\frac{1}{2} x^2} y) = x e^{\frac{1}{2} x^2}$$

Integrate

$$e^{\frac{1}{2} x^2} y = e^{\frac{1}{2} x^2} + C$$

$$y = 1 + C e^{-\frac{1}{2} x^2}$$

Replace

$$6 = 1 + C$$

$$5 = C$$

Answer

$$y = 1 + 5 e^{-\frac{1}{2} x^2}$$