

The slope of the tangent line to a curve $y = f(x)$ at the point $P_0(0,1)$ is equal to $\frac{1}{2}$.

Find the equation of the curve given that it satisfies the second order differential

equation $\frac{d^2y}{dx^2} = x^3 + \sin x$

Solution:

Integrate once to get $y' = \frac{1}{4}x^4 - \cos x + C$

Since $y'(0) = \frac{1}{2}$, we conclude that $C = \frac{3}{2}$, therefore

$$y' = \frac{1}{4}x^4 - \cos x + \frac{3}{2}$$

Integrate a second time, $y = \frac{1}{20}x^5 - \sin x + \frac{3}{2}x + C_1$

Since $y(0) = 1$, we conclude that $C_1 = 1$, therefore

$$y = \frac{1}{20}x^5 - \sin x + \frac{3}{2}x + 1$$

Answer

$$y = \frac{1}{20}x^5 - \sin x + \frac{3}{2}x + 1$$