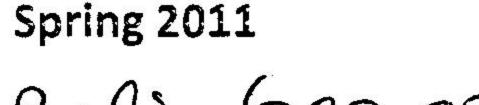Haigazian University
MATH 219 Test 3
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1. Given the matrix

$$A = \begin{bmatrix} 1 & 1 & a \\ 1 & a & 1 \\ a & 1 & 1 \end{bmatrix}$$

   (a) What is $rank(A)$ when $a = 1$?
   (b) Find the value of a for which $rank(A) = 2$

2. (a) Find a basis for the nullspace of the matrix

$$B = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

   (b) Find a basis for the row space of B

3. What conditions must be satisfied by b_1, b_2, b_3, b_4, and b_5 for the following
   linear system to be consistent?

$$\begin{aligned} x_1 - 3x_2 &= b_1 \\ x_1 - 2x_2 &= b_2 \\ x_1 + x_2 &= b_3 \\ x_1 - 4x_2 &= b_4 \\ x_1 + 5x_2 &= b_5 \end{aligned}$$

4. Fill in the blank.
   (a) If A is a 3×5 matrix, then the number of leading 1's in the reduced row-
       echelon form of A is at most ___3___
   (b) If A is a 3×5 matrix, then the number of free variables (parameters) in
       the general solution of $Ax = 0$ is at least ___2___
   (c) The vector space of all diagonal $n \times n$ matrices has dimension ___n___
   (d) If A is a 5×3 matrix, then the nullity of A^T is at least ___2___

5. Let $p_1 = 1 + x$, $p_2 = 1 + x^2$, $p_3 = x + x^2$
   (a) Show that $S = \{p_1, p_2, p_3\}$ is a basis for P_2
   (b) Express $p = 2 - x + x^2$ as a linear combination of p_1, p_2, p_3

6. Given the three matrices

$$A = \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}$$

   Find a matrix D such that $\{A, B, C, D\}$ is a basis for M_{22}