

1. Let

$$A = \begin{bmatrix} 3 & 4 \\ -1 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 5 \\ 3 & 0 \end{bmatrix}$$

Calculate

$$\text{tr}(A), \quad \text{tr}(B), \quad \text{tr}(AB), \quad \text{tr}(AA^T)$$

2. A system of 3 equations in 3 unknowns $\{x_1, x_2, x_3\}$ has the following augmented matrix

$$G = \begin{bmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{bmatrix}$$

where a, b are given real numbers.

- If $a \neq 0$, find the reduced row-echelon form of G and solve the system.
- How many solutions does the system have if $a = 0$?

3. Use elementary row operations to find the inverse of the matrix

$$M = \begin{bmatrix} 1 & 2a & 10ac \\ 0 & 2 & 10c \\ 0 & 0 & 1 \end{bmatrix}$$

where a, c are given real numbers.

4. If the size of the matrix A is $m \times n$ and $A(BA)$ is defined, what is the size of the matrix B ?

5. Given

$$\left(I_2 + \frac{1}{8}A\right)^{-1} = \begin{bmatrix} -1 & 2 \\ -1/2 & 5 \end{bmatrix}$$

find A

6. Suppose A and $A + B$ are both invertible, and let C be the inverse of $A + B$. Show that

$$\begin{aligned} \text{aa) } & (I + BA^{-1})AC = I \\ \text{b) } & AC(I + BA^{-1}) = I \end{aligned}$$