HAIGAZIAN UNIVERSITY **MATH 219 FINAL EXAM** **Spring 2011**

Instructor: Ara Jamboulian

Date: Wed 15 June 2011

Time: 8:00 - 10:30 am

Room: C-601

1. Use elementary row operations to find the determinant of the matrix

$$\begin{bmatrix} -2 & 3 & 2 & 1 \\ 6 & 0 & 3 & 3 \\ 4 & 1 & -1 & 0 \\ 2 & 4 & 0 & 0 \end{bmatrix}$$

2. Find the rank and the nullity of the matrix

$$A = \begin{bmatrix} 1 & 8 & 0 & 2 \\ 2 & 2 & 4 & 0 \\ 2 & 9 & 2 & 2 \end{bmatrix}$$

3. Consider the 3×3 matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ m & 2 & 0 \\ b & 1 & -1 \end{bmatrix}$$

where m and b are real numbers.

a) Find the characteristic polynomial and the eigenvalues of A

b) Find the eigenvectors of A

c) Is A diagonalizable?

4. Let $v_1 = (1, -4, 2, -3)$ and $v_2 = (-3, 8, -4, 6)$. Find two vectors v_3, v_4 such that $\{v_1, v_2, v_3, v_4\}$ is a basis for R^4

5. Let A be a 2×2 matrix with $\mathrm{tr}(A) = 2$ and $\det(A) = -3$. Show that A is diagonalizable

6. Let $T: R^3 \to R^3$ be the linear transformation defined by $T(v) = A.v$ where

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & 6 & -4 \\ 7 & 4 & 2 \end{bmatrix}$$

a) Find a basis for the range of T

b) Find a vector $\boldsymbol{w}$ which is not in the range of T

7. Let $\boldsymbol{W}$ be the set of polynomials $p(x) = a_0 + a_1 x + a_2 x^2$ in P_2 such that $p(1) = 0$.

a) Show that $\boldsymbol{W}$ is a subspace of P_2

b) Find $\dim \mathbf{W}$

8. Find a square matrix B such that $tr(B) = \det(B) = 4$ and B is not diagonalizable

9. Let T be the linear operator on R^2 defined by $T(x_1, x_2) = (-x_2, x_1)$

a) Write down the matrix of T in the ordered basis $\mathfrak{B} = \{w_1, w_2\}$ where

$$w_1 = (1,2), \quad w_2 = (1,-1)$$

b) Show that T is one-to-one and give a formula for $T^{-1}(x_1, x_2)$