Haigazian University **Spring 2011**

MATH 219 Test 4 Solution

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1. Let $S = \{v_1 , v_2\}$ where $v_1 = (1,1)$, $v_2 = (-1,1)$

(a) Suppose $c_1 v_1 + c_2 v_2 = (0,0)$

$c_1 - c_2 = 0$ and $c_1 + c_2 = 0 \implies c_1 = c_2 = 0$

(b) $(5,6) = \frac{11}{2}\, v_1 + \frac{1}{2}\, v_2$

$$T(5,6) = \frac{11}{2}\, T(v_1) + \frac{1}{2}\, T(v_2) = \frac{11}{2}\, (2,3) + \frac{1}{2}\, (3,4) = \left(\frac{25}{2}, \frac{37}{2}\right)$$

2. Let $T: R^2 \rightarrow R^3$ be the linear transformation given by the formula

$$T(x_1 , x_2) = (x_1 - x_2 , \, x_2 - x_1 , -x_1)$$

(a) $T(1,0) = (1,-1,-1)$ and $T(0,1) = (-1,1,0)$ are linearly independent therefore $\mathfrak{B} = \{(1,-1,-1), (-1,1,0)\}$ is a basis for the range of T

(b) rank of $T = 2$

(c) nullity of $T = 0$

3.

$$A\boldsymbol{v} = \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} -6 \\ 8 \\ 3 \end{bmatrix} = \begin{bmatrix} 24 \\ -32 \\ -12 \end{bmatrix} = -4 \begin{bmatrix} -6 \\ 8 \\ 3 \end{bmatrix} = -4\boldsymbol{v}$$

Characteristic polynomial

$x^3 - 2x^2 - 15x + 36 = (x+4)(x-3)^2$

Eigenvalues of A : $-4, 3$

The eigenspace E_3 has dimension 1. So A is not diagonalizable

4. Let

$$B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

Eigenvalues: 1 , 2

Eigenvectors: (1,1) and (0,1)

$P = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$P^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

$P^{-1}BP = D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$B = P\, D\, P^{-1}$

$B^{10} = P\, D^{10} P^{-1} = P \begin{bmatrix} 1^{10} & 0 \\ 0 & 2^{10} \end{bmatrix} P^{-1} = \begin{bmatrix} 1 & 0 \\ -1023 & 1024 \end{bmatrix}$