

1. Let  $S = \{v_1, v_2\}$  where  $v_1 = (1,1)$ ,  $v_2 = (-1,1)$

(a) Show that  $S$  is a basis for  $\mathbb{R}^2$

(b) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation given by

$$T(v_1) = (2,3)$$

$$T(v_2) = (3,4)$$

Compute  $T(5,6)$

2. Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be the linear transformation given by the formula

$$T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1)$$

(a) Find a basis for the range of  $T$

(b) What is the rank of  $T$ ?

(c) What is the nullity of  $T$ ?

3. Given the matrix

$$A = \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix}$$

and the vector

$$v = \begin{bmatrix} -6 \\ 8 \\ 3 \end{bmatrix}$$

(a) Calculate the product  $Av$  and show that  $v$  is an eigenvector of  $A$

(b) Find the characteristic polynomial and the eigenvalues of  $A$

(c) Show that  $A$  is not diagonalizable

4. Let

$$B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

(a) Find an invertible matrix  $P$  that diagonalizes the matrix  $B$

(b) Compute  $B^{10}$

(7-1)

$$\begin{aligned} 5x_1 + 6x_2 + 2x_3 &= 0 \\ -x_2 - 8x_3 &= 0 \\ -2x_3 &= 0 \end{aligned}$$

$x_1$