Math 219 Test 3 Solution

1. (a) When $a = 1$ the reduced row echelon form of the matrix A is

$$R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore $rank\ A = rank\ R = 1$.

(b) Reduce A to a row echelon form assuming $a \neq 1$. The result is

$$B = \begin{bmatrix} 1 & 0 & a+1 \\ 0 & 1 & -1 \\ 0 & 0 & a+2 \end{bmatrix}$$

$rank\ A = rank\ B = 2$ implies $a + 2 = 0$. Therefore $a = -2$

2. (a) The reduced row echelon form of B is

$$R = \begin{bmatrix} 1 & 0 & 1 & -2/7 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Choose two free variables $x_4 = t\ and\ x_3 = s$. The general solution is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2/7 \\ -4/7 \\ 0 \\ 1 \end{bmatrix}$$

A basis for the null space of B is given by $\left\{(-1, -1, 1, 0), \left(\frac{2}{7}, -\frac{4}{7}, 0, 1\right)\right\}$

(b) A basis for the row space of B : $\{ \left(1, 0, 1, -\frac{2}{7}\right), \left(0, 1, 1, \frac{4}{7}\right) \}$

3. This is HW problem 9 section 5.6 p 270. **Answer**:

$$b_1 = r, b_2 = s, b_3 = 4s - 3r, b_4 = 2r - s, b_5 = 8s - 7r$$

4. 3,2,n,2

5. (a) Suppose $c_1p_1 + c_2p_2 + c_3p_3 = 0$

$$c_1(1 + x) + c_2(1 + x^2) + c_3(x + x^2) = 0$$
$$(c_1 + c_2) + (c_1 + c_3)x + (c_2 + c_3)x^2) = 0$$

$$c_1 + c_2 = 0 \quad c_1 + c_3 = 0 \quad c_2 + c_3 = 0$$
$$c_1 = c_2 = c_3 = 0$$

(b) $p = 2p_2 - p_3$

6. A matrix $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is in span$\{A, B, C\}$ if and only if

$$\det \begin{bmatrix} 3 & 0 & 0 & a \\ 6 & -1 & -8 & b \\ 3 & -1 & -12 & c \\ -6 & 0 & -4 & d \end{bmatrix} = 0$$

$$a + b - c + d = 0$$

Answer: choose any matrix D such that $a + b - c + d \neq 0$