MATHEMATICS 201
FIRST SEMESTER, 2006-07
QUIZ II

Time: 70 Minutes.

Date: DECEMBER 9, 2006.

Name:—————————————-

ID Number:———————————

Section:————————————-

Circle Section Number:

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Instructions: The examination consists of two parts: Part I consists of **five written questions**, and Part II of **five multiple-choice questions**. Please observe the following:

(a) Answers of questions of Part I must be fully justified.

(b) Circle only one choice (a), (b), $\cdots$, or (e) for each question of Part II.

(c) Each multiple-choice question has exactly one answer.

(d) Circling none or more than one answer for a question of Part II results in a zero credit.

(e) The grade allocated to each question is set next to it.

GRADE OF PART I:	/75
GRADE OF PART II:	/25
TOTAL GRADE:	/100

PART I:

(1) Consider the polar curves $r = 3 + 2\cos\theta$ and $r = 2$.

(a) Sketch the curves. (6 points)

(b) Find their points of intersection. (6 points)

(c) Shade the region that lies inside the curve $r = 2$ and outside the curve $r = 3 + 2\cos\theta$ and find its area.

(8 points)

(2) Consider the $2\pi-$periodic function

$$f(x) = \begin{cases} x, & \text{if } 0 \le x \le \pi \\ 0, & \text{if } \pi < x \le 2\pi. \end{cases}$$

(a) Show that the Fourier series of f is (10 points)

$$\frac{\pi}{4} - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx.$$

(b) Show that (5 points)

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}.$$

(3) (a) Use the Maclaurin series of $\cos x$ to find the Maclaurin series for $x^2 \cos x^2$. (5 points)

(b) Find the polynomial that approximates the function (8 points)

$$f(x) = \int_0^x t^2 \cos t^2 \, dt$$

for all values x, $|x| \leq 0.1$, so that the error of approximation is at most 10^{-6}.

(c) Evaluate $f^{(2k+1)}(0)$ for all $k = 1, 2, \cdots$. (5 points)

(4) Consider the function

$$f(x, y, z) = \frac{1}{\sqrt{25 - x^2 - y^2 + z^2}}.$$

(a) Find the domain and range of f. (4 points)

(b) Find the boundary of the domain of f. (2 points)

(c) State whether the domain is open, closed, or bounded. Justify. (4 points)

(d) Find an equation for the level surfaces of f. (3 points)

(e) Sketch the level surface of f that passes through $(3, 4, 5)$. (3 points)

(5) Show that the function

$$f(x,y) = \begin{cases} \dfrac{x^4 y}{x^6 + y^3}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is not continuous $(0,0)$. (6 points each)

PART II: Multiple-choice questions: (5 points each)

(6) The fourth term of the Maclaurin series of $\sqrt[3]{1+2x}$ is

(a) $50x^3/3^4$.

(b) $48x^3/3^3$.

(c) $40x^3/3^4$.

(d) $64x^3/3^3$.

(e) None of the above.

(7) The Taylor remainder of order 3 obtained from the Taylor Estimation theorem in the approximation $e^x = 1 + x + (x^2/2)$ for values $|x| < 0.1$ is at most

(a) $e^{-0.1}/3000$.

(b) $e^{0.001}/60000$.

(c) $e^{0.1}/600$.

(d) $e^{0.1}/6000$.

(e) None of the above.

(8) The slope of the tangent line to the polar curve $r = \sin 3\theta$ at $\theta = \pi/3$ is

(a) 1.

(b) $\sqrt{3}$.

(c) $-\sqrt{3}$.

(d) $\sqrt{3}/3$.

(e) None of the above.

(9) The value of the limit

$$\lim_{x\to 0}\left[\frac{x\ln(1+x)+2\cos x-2+x^3/2}{x^4}\right]$$

is

(a) 5/12.

(b) 4/11.

(c) 3/10.

(d) 2/7.

(e) None of the above.

(10) The limit

$$\lim_{(x,y)\to(0,0)} \frac{[\cos(x^2+y^2)]-1}{(x^2+y^2)^2}$$

equals

(a) 0.

(b) 1.

(c) -1.

(d) Does not exist.

(e) None of the above.