I. (20 pts- 5 pts each)

a) Find the Maclaurin series for e^x.

b) Find the radius and interval of convergence of the Maclaurin series for e^x.

c) Find the Maclaurin series for $\frac{1}{e}$?

d) Evaluate $\frac{1}{e}$ with an error of magnitude less than $\frac{1}{7!}$?

II. (10 pts – 5 pts each) Consider the polar curve $r^2 = \cos 2\theta$

a) Sketch the curve of the lemniscates.

b) Find the area enclosed by one loop of the above lemniscate.

III. (5 pts) Sketch the following surface :

$$2y^2 + 3z^2 + x^2 = 18$$

IV. (10 pts) Find the volume which lies inside of the sphere centered at the origin and of radius19 and between the cones $\varphi = \frac{\pi}{4}$ & $\varphi = \frac{\pi}{6}$

V. (10pts) Find all local maxima, minima, saddle points of the equation:

$$f(x,y) = 5x^2y - 2xy^2 + 30xy - 3$$

VI. (5 pts each) Evaluate the following integral :

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\,dx\,dy$$

VII. (10 pts) Evaluate the volume of the region in space bounded by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$

VIII. (10 pts) Find the spherical coordinate limits for the integral that calculates the volume of the solid formed by the intersection of the inside of the sphere $(y-3)^2 + z^2 + x^2 = 25$ and the inside of the cylinder $y^2 + x^2 = 4$

IX. (5 pts) Find the directional derivative of $f(x,y,z) = \cos(xy) + z^2$ at $p_0\left(1, \frac{\pi}{2}, 1\right)$ in the direction of vector $\vec{v} = 4\vec{i} + -3\vec{k}$

X. (5 pts) Maximize the function

$f(x,y,z) = xyz$ subject to the constraints $2xy + 2yz + 2xz = 64$

XI. (5pts) Find the Fourier series of $f(x) = 2x + 4 \qquad where\ -\pi \le x \le \pi$

XII. (5pts) Find using the Maclaurin series the following limit:

$$\lim_{x \to 0} \frac{(\sin x - x)^3}{x^9}$$