



1. (10 pts - 5 pts each) Which of the following sequences converge and which diverge? Find the limit of each convergent sequence.

a)  $a_n = \left\{ \frac{n!}{(2n)!} \right\}^2$

b)  $a_n = \left\{ \sin \frac{1}{n} \right\}$

2. (15 pts - 5 pts each) Which of the following series converges and which diverges. Find the sum of each converging series if possible:

a)  $\sum_{n=5}^{\infty} \frac{1}{(2 + \pi)^{2n}}$

b)  $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{1 + n\sqrt{n}}$

c)  $\sum_{n=4}^{\infty} \frac{n^{100} 2^n}{\sqrt{n!}}$

3. (10 pts) Check whether the series converges absolutely, converges conditionally or diverges?

$$\sum_{n=1}^{\infty} \frac{\left[ \sin \frac{(2n+1)\pi}{2} \right]}{\ln(\ln n)}$$

4. (10 pts) Find the series' radius and interval of convergence and identify the values of  $x$  for which the series converges absolutely and conditionally.

$$\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{\sqrt{n+1}}$$

5. (10 pts) Use the power series (Maclaurin Series) to evaluate the limit.

$$\lim_{x \rightarrow 0} \frac{(e^x - 1 - x)^2}{x^2 - \ln(1 + x^2)}$$



6. (10 pts) How many terms are needed to approximate the sum of the series

$\sum_{x=1}^{\infty} \tan^{-1} x$  with an error of magnitude no greater than  $10^{-4}$ ? Give reasons for your answers, and estimate the sum of the series.

7. (5 pts) Find  $L(0.5)$  with an error of magnitude less than  $10^{-3}$ , knowing that

$$L(x) = \int_0^x \frac{e^t - 1}{t} dt$$

8. (10 pts) Find the Fourier series of the function on the given interval:

$$f(t) = \begin{cases} 0 & -1 \leq t \leq 0 \\ t & 0 \leq t < 1 \end{cases}$$

9. (10pts-5 pts each)

a) Change the following equation from polar coordinates to Cartesian coordinates:  $r = 1 - 2 \sin \theta$

b) Change the following equation from Cartesian coordinates to polar coordinates:  $x + y^2 = 2y + 3$

10. (10pts-5pts each)

a) Graph :  $r = \sin 3\theta$ .

b) Graph :  $r = 2 \cos \theta$ .