I. **(30pts- 10pts each)** Solve the following :

   a. Find the area of the region that lies inside the cardiod $r = 1 + \sin\theta$ and outside the circle $r = 1$

   b. Find the line tangent to the curve: $r = \cos 2\theta$ at $\theta = \dfrac{\pi}{3}$

   c. Find the length of the curve $r = \cos^3\left(\dfrac{\theta}{3}\right)$ when $0 \le \theta \le \dfrac{\pi}{4}$

II. **(20pts- 10pts each)** Given $f(x, y, z) = \left(\ln \sqrt{x^2 + 4y}\right)\left(z^2 + 5xy\right)$.

   a. Find the directional derivative of the above function at the point $P(1,2,1)$ in the direction of $\vec{v} = 2\vec{i} + 4\vec{j} - \vec{k}$

   b. Find the direction from the point $P(1,2,1)$ in which the above function increses most rapidly

III. **(10pts)** Find and classify the critical points (maxima, minima, and saddle points) of the function $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$

IV. **(10pts)** Find the extreme values of the function $f(x, y, z) = x^2 + 2y - z^2$ subject to the constraint $2x + y - 3z = 0$

V. **(10pts)** Determine and graph the level curves of the following function:
$f(x, y) = 50 - 2x^2 - 2y^2$

VI. **(10pts)** Find $\dfrac{\partial f}{\partial x}\Big|_{(x, y)=(\pi,1)}$ if $f(x, y) = 4\sin y - \dfrac{2}{\sqrt{xy}}$

VII. **(10pts)** Find Find $\dfrac{\partial^2 w}{\partial y \partial x}$ if $w = x\sin y + e^{y^2}$

BONUS: (10pts)

Graph the lemniscate: $r^2 = \cos 2\theta$