American University of Science and Technology

MATHEMATICS DEPARTMENT
MAT 102/202: CALCULUS II
(VERSION B)

EXAM II

FALL 2009-2010

Student's Name: Answer Key

ID #: ____________________

Instructor's Name: ____________________

Section: ____________________

Please be sure that you read and understand the following points:

- The CHEATING penalty will be " F " on the course
- Time duration is 60 minutes
- No examination booklets are needed for this exam
- Non-Programmable calculators are allowed
- All documents and notes are NOT allowed
- Total number of sheets including this is 6

Show details of your work. Writing answers with no details shall cost you a loss of 50% of a question/problem grade. No questions are allowed. If something is not understood, write your assumptions and solve the problem.

1. (80 pts – 10 pts each) Solve the following integrals

a) $\int_{0.5}^{1} \frac{dx}{3x - x^2}$

$$\frac{1}{3x-x^2} = \frac{1}{x(3-x)} = \frac{A}{x} + \frac{B}{3-x}$$

$$A = B = \frac{1}{3}$$

$$I = \int_{0.5}^{1} \frac{\frac{1}{3}dx}{x} + \int_{0.5}^{1} \frac{\frac{1}{3}dx}{3-x} = \frac{1}{3}\ln|x| + -\frac{1}{3}\ln|3-x| \Big]_{0.5}^{1}$$

$$= \frac{1}{3}\ln\left|\frac{x}{3-x}\right| \Big]_{0.5}^{1} = \frac{1}{3}\ln\left(\frac{1}{2}\right) - \frac{1}{3}\ln\left(\frac{1}{5}\right)$$

$$= \frac{1}{3}\ln\left(\frac{\frac{1}{2}}{\frac{1}{5}}\right) = \boxed{\frac{1}{3}\ln\left(\frac{5}{2}\right)}$$

b) $\int \frac{dx}{2\sqrt{x} + 2x} = \int \frac{dx}{2\sqrt{x}\,(1+\sqrt{x})}$

Let $u = 1 + \sqrt{x}$
$du = \frac{dx}{2\sqrt{x}}$

$$I = \int \frac{du}{u} = \ln|u| + k$$

$$= \boxed{\ln(1+\sqrt{x}) + k}$$

c) $\int \frac{x^3+2x^2-x+1}{x^2+1}\,dx$

$$\begin{array}{r|l} & x+2 \\ \hline x^2+1 & x^3+2x^2-x+1 \\ & -x^3 \quad\quad -x \\ \hline & 2x^2-2x+1 \\ & -2x^2 \quad\quad -2 \\ \hline & -2x-1 \end{array}$$

$$I=\int\left(x+2+\frac{-2x-1}{x^2+1}\right)dx$$

$$I=\frac{x^2}{2}+2x-\int\frac{2x\,dx}{x^2+1}-\int\frac{dx}{x^2+1}$$

$$\boxed{I=\frac{x^2}{2}+2x-\ln(x^2+1)-\tan^{-1}x+k}$$

d) $\int \frac{3\ln x\,dx}{x(1-2\ln^2 x)}$

Let $u=1-2\ln^2 x$

$du=-4\ln x\cdot\frac{1}{x}dx$

$\frac{du}{-4}=\frac{\ln x}{x}dx$

$$I=\int\frac{-\frac{3}{4}du}{u}=\frac{-3}{4}\ln(u)+k$$

$$\boxed{I=-\frac{3}{4}\ln|1-2\ln^2 x|+k}$$

e) $\int \frac{dx}{x^2+5x+6}$

$$\int \frac{dx}{x^2+5x+6} \Rightarrow \frac{1}{x^2+5x+6} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$\frac{1}{x^2+5x+6} = \frac{A(x+3)+B(x+2)}{(x+2)(x+3)}$$

$$A=1\ ,\ B=-1$$

$$I = \int \frac{1}{x+2}\,dx + \int \frac{-1}{x+3}\,dx$$

$$\boxed{I = \ln|x+2| - \ln|x+3| + k \qquad I = \ln\left|\frac{x+2}{x+3}\right| + k}$$

f) $\int \frac{2-3x}{\sqrt{4-x^2}}\,dx$

$$I = \int \frac{2}{\sqrt{4-x^2}}\,dx - 3\int \frac{x\,dx}{\sqrt{4-x^2}} = \int \frac{dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}} - 3\int \frac{x\,dx}{\sqrt{4-x^2}}$$

$$= \boxed{2\sin^{-1}\left(\frac{x}{2}\right) - 3\sqrt{4-x^2} + k}$$

g) $\int (x^3+2x-1)e^{2x}\,dx$

x^3+2x-1	$+$	e^{2x}
$3x^2+2$	$-$	$e^{2x}/2$
$6x$	$+$	$e^{2x}/4$
6	$-$	$e^{2x}/8$
0		$e^{2x}/16$

$$\boxed{I=\frac{(x^3+2x-1)}{2}e^{2x}-\frac{(3x^2+2)}{4}e^{2x}+\frac{6x}{8}e^{2x}-\frac{6}{16}e^{2x}+k}$$

h) $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

$4-x^2=2^2-x^2 \quad a^2-u^2$

Let $u=a\sin\alpha$

$\Rightarrow$ Let $x=2\sin\alpha \Rightarrow dx=2\cos\alpha\,d\alpha$

$4-x^2=4-4\sin^2\alpha=4(1-\sin^2\alpha)$

$=4\cos^2\alpha$

$\sqrt{4-x^2}=2\cos\alpha$

$$I=\int\frac{4\sin^2\alpha\cdot\cancel{2\cos\alpha}\,d\alpha}{\cancel{2\cos\alpha}}=\int 4\sin^2\alpha\,d\alpha=\int 4\left(\frac{1-\cos 2\alpha}{2}\right)d\alpha$$

$$I=\int 2(1-\cos 2\alpha)\,d\alpha$$

$$I=2\left[\alpha-\frac{\sin 2\alpha}{2}\right]=2\left[\alpha-\sin\alpha\cos\alpha\right]$$

$$\boxed{I=2\left[\sin^{-1}\left(\frac{x}{2}\right)-\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right)\right]+k}$$

2. (20pts) $(\sinh x)^2 = \frac{1}{4},\ x > 0$ $\Rightarrow \boxed{\sinh x = \pm\frac{1}{2}}$

a) Find $\cosh x$, $\operatorname{csch} x$, $\tanh x$, *and* $\operatorname{sech} x$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh^2 x - \frac{1}{4} = 1 \Rightarrow \cosh^2 x = \frac{5}{4} \Rightarrow \boxed{\cosh x = \frac{\pm\sqrt{5}}{2}}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\pm 1/2}{\pm\sqrt{5}/2} = \boxed{\pm\frac{1}{\sqrt{5}}}$$

$$\coth x = \frac{1}{\tanh x} = \boxed{\pm\sqrt{5}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \boxed{\pm\frac{2}{\sqrt{5}}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \boxed{\pm 2}$$

b) Solve for x $\quad \frac{e^x - e^{-x}}{2} = \begin{cases} +\frac{1}{2} \\ -\frac{1}{2} \end{cases}$

$$\frac{e^x - e^{-x}}{\cancel{2}} = \frac{1}{\cancel{2}} \Rightarrow e^x - \frac{1}{e^x} = 1 \Rightarrow e^{2x} - 1 = e^x$$

$$e^{2x} - e^x - 1 = 0 \qquad \Delta = (-1)^2 - 4(1)(-1) = 5$$

$$e^x = \frac{1+\sqrt{5}}{2(1)} \Rightarrow \boxed{x = \ln\left(\frac{1+\sqrt{5}}{2}\right)}$$

$$\cancel{e^x = \frac{1-\sqrt{5}}{2}} \rightarrow \text{Rejected.}$$