

1- Main Formula:

$$V = \int_a^b A(x) dx$$

This formula is applied in the following 3 methods:

i- Slicing Method:

Take a cross-sectional slice of the shape.

A(x) is found depending on the shape of the cross-section.

Example: Rectangle: $A(x) = \text{length} * \text{width}$

Circle: $A(x) = \pi R^2$

ii- Disk Method:

The cross-section is already known to be a circle.

so $A(x) = \pi R^2$

$$\text{and } V = \int_a^b \pi R^2 dx$$

iii- Washers Method:

Donut-shaped cross-sections, so the cross-section area is:

Area of Outer Circle - Area of Inner Circle

$$\underline{A(x) = \pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2}$$

$$\text{and so, } V = \int_a^b \pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2 dx$$

$$\text{that is, } V = \int_a^b \pi (R_{\text{outer}}^2 - R_{\text{inner}}^2) dx$$

2- Cylindrical Shells Formula:

$$\text{and } V = \int_a^b \text{Circumference} * \text{height} dx$$

$$\text{and } V = \int_a^b 2 \pi R * \text{height} dx$$