Math 201 — Fall 2004–05
Calculus and Analytic Geometry III, sections 5–8
Quiz 2, December 2 — Duration: 1 hour

GRADES (each problem is worth 12 points):

1	2	3	4	5	6	TOTAL/72

YOUR NAME: Correction to Quiz II

YOUR AUB ID#:

PLEASE CIRCLE YOUR SECTION:

Section 5 Recitation M 1 Professor Makdisi	Section 6 Recitation Tu 12:30 Mr. Khatchadourian	Section 7 Recitation Tu 2 Mr. Khatchadourian	Section 8 Recitation Tu 3:30 Mr. Khatchadourian

INSTRUCTIONS:

1. Write your NAME and AUB ID number, and circle your SECTION above.
2. Solve the problems inside the booklet. Explain your steps precisely and clearly to ensure full credit. Partial solutions will receive partial credit. Each problem is worth 12 points.
3. You may use the back of each page for scratchwork OR for solutions. There are three extra blank sheets at the end, for extra scratchwork or solutions. If you need to continue a solution on another page, INDICATE CLEARLY WHERE THE GRADER SHOULD CONTINUE READING.
4. No calculators, books, or notes allowed. Turn OFF and put away any cell phones.

GOOD LUCK!

An overview of the exam problems. Each problem is worth 12 points.
Take a minute to look at all the questions, THEN
solve each problem on its corresponding page INSIDE the booklet.

1. Some multiple-choice questions on the next page: (i) relate surfaces and their equations, (ii) approximate the values of a function, (iii) questions about level curves. Note that there are SIX parts 1a–1f, with 2 points for each part.

2. Using polar coordinates, **sketch and clearly label** the curves C_1, C_2 given by

$$C_1 : r = \cos\theta, \qquad C_2 : r = 1 - \cos\theta$$

and **find the area** of the region that is **inside** C_1 **and outside** C_2.

3. Consider the function $f(x,y,z) = ye^{x^2 z}$ and the point $P_0(-1,2,1)$.
a) Find the gradient $\vec{\nabla} f \Big|_{P_0}$.
b) Find the equation of the tangent plane to the surface $ye^{x^2 z} = 2e$ at the point P_0.
c) Find the directional derivative of f at P_0 in the direction of the vector $\vec{v} = (1,2,3)$.

4. Given the parametrized curve in space: $P(t) = (x(t), y(t), z(t)) = (e^t, e^{-t}, \sqrt{2} \cdot t)$.
a) Find the arclength of the part of the curve between the points $P_1 = (1,1,0)$ (corresponding to $t_1 = 0$) and $P_2 = (e, 1/e, \sqrt{2})$ (corresponding to $t_2 = 1$). Note: you can, and should, simplify the expression inside the integral to get rid of the square root.
b) (UNRELATED to part (a)) Use the same curve $P(t)$ as above. Given a function $f(P) = f(x,y,z)$ with $\vec{\nabla} f \Big|_{(3,1/3,\sqrt{2}\ln 3)} = (1,6,4)$, find the derivative $\frac{d}{dt}\Big[f(P(t))\Big]\Big|_{t=\ln 3}$.

5. a) Find the maximum and minimum values of the function $f(P) = f(x,y,z) = x+y+z$, under the **constraint** that the point $P(x,y,z)$ is restricted to lie on the ellipsoid $2x^2 + y^2 + z^2 = 1/2$.
b) (UNRELATED to part (a)) Find the critical points of the function $f(x,y) = x^3 - x + 2xy + y^2$, and classify each point as a local minimum, a local maximum, or a saddle point.

6. a) Using the two-path test, show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^4 + y^4}$ does NOT exist.
b) (Challenging) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y^2}{x^4 + y^4} = 0$.

7. Let $f(x, y) = e^{-3xy+5}$. Then its critical point is

A) a local min.
B) a local max.
C) a saddle point.

8. Consider the paraboloid $x^2 + y^2 - 4z = 1$ and the sphere $x^2 + y^2 + z^2 = 3$. Then the ***tangent planes*** to both surfaces at the intersection point (1, 1, 1) are

A) parallel
B) perpendicular
C) neither perpendicular nor parallel.

9. Given that $F(x, y, z) = 8$. If the components of ∇F are never zero, then

$$\frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \quad \& \quad \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial y} \quad \text{are}$$

A) -1 & $-\frac{\partial z}{\partial y}$ resp. B) $+1$ & $-\frac{\partial z}{\partial y}$ resp.

C) -1 & $\frac{\partial z}{\partial y}$ resp. D). $+1$ & $\frac{\partial z}{\partial y}$ resp. E) None of the above

10. The value of the double integral $\int_0^2 \int_{y/2}^1 3ye^{x^3}\, dx dy$ is

A) 9/2 (e-1)
B) 2(e-1)
C) 8(e-1)
D) 25/2 (e-1)

Part II (50 %) (Subjective)

11. (5 %) Find the area of the surface cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane $z = 9$. (Grading: 4pts for setting it up & changing it to polar)

12. (7 %) Use Green's Theorem to find $\oint_C xy^3dx + (2x^2y^2 + 1)dy$

where C (traversed counterclock wise) is the boundary of the "triangular region in the 1^{st} quadrant enclosed by the x-axis, x=1 and $y = x^2$

13. (5 %) Set up (*<u>but do not evaluate</u>*) the double integral(s) in **<u>polar</u>** coordinates to find the area of the "triangular" region in the first quadrant bounded by $y=4x^2$, $x=0$ & $x+y = 5$.. **<u>Hint</u>**: The point (1, 4) is a corner point of the region.

14. (8 %)

(i) Show that $\mathsf{F}=(y-x^2)\mathsf{i}+(x+y^2)\mathsf{j}$ is a conservative vector field

(ii) Find a potential function for **F**

(iii) Evaluate $\oint_C (y-x^2)\,dx+(x+y^2)\,dy$ where C is the line segment from (0, 1) to ((3,0).

15. (5 %) Set up (*but do not evaluate*) the triple integral(s) in **Spherical** coordinates to find the volume and in the first octant of the surface inside the cylinder $x^2 + y^2 = 4$ and inside the sphere $x^2 + y^2 + z^2 = 8$

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16. (5%) Set up (*but do not evaluate*) the triple integral(s) in **Cylinderical** coordinates to find the volume in the 1st octant common to the cylinders $x^2 + y^2 = 4$ and $4x^2 + z^2 = 1$.

17. (5 %) Consider the transformation $u = x - xy$ & $v = xy$
(so $x = u + v$ & $y = ...$)

(i) Show that the Jacobian $J = \dfrac{\partial(x,y)}{\partial(u,v)} = \dfrac{1}{u+v}$.

(ii) Use the above transformation to find $\iint_R x\,dydx$ where R is the region bounded by the curves $x - xy = 1,\ x - xy = 2,\ xy = 1,\ xy = 3$