Quiz II Math 201 A-Lyzzaik (2) B

An overview of the exam problems
(the problems are repeated inside the booklet).

1. a) *(4 points)* Sketch the following curves in polar coordinates and find the two intersection points. There are no tricks involved!!!

$$C_1 : r = \frac{3}{2} \qquad C_2 : r = 1 + \cos\theta$$

b) *(4 points)* Using polar coordinates, SET UP an integral which gives the area of the region that is *outside* C_1 and *inside* C_2. DO NOT EVALUATE THE INTEGRAL.

2. *(8 points)* Find the equation of the tangent plane to the surface $z = x^2 + 3y^2 - 7$ at the point $(2, -1, 0)$.

3. Consider the function

$$f(x, y, z) = z + \sqrt{4 - x^2 - y^2}.$$

a) *(2 points)* Describe the domain of f and sketch a picture.
b) *(3 points)* Is the domain open? Why or why not?
c) *(3 points)* Is the domain bounded? Why or why not?

4. *(8 points)* Prove that the function

$$f(x, y) = \begin{cases} x \cdot \sin \dfrac{1}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous at $(0, 0)$.

5. Let R be the region in the xy-plane *between* the line $y = 2x$ and the parabola $y = x^2$.
a) *(8 points)* Write down the iterated integrals for $\int\int_R x \, dA$ in *both* orders (this means both $dy\, dx$ and $dx\, dy$).
b) *(4 points)* Evaluate ONE of the integrals from part a).

6. We are given a differentiable function $f(x, y)$ such that:

$$\vec{\nabla} f(0, 1) = -\vec{i} + \vec{j} \qquad \vec{\nabla} f(1, -4) = 2\vec{i} + 3\vec{j} \qquad \vec{\nabla} f(1, 2) = \vec{i} + \vec{j}.$$

a) *(8 points)* By about how much will $f(x, y)$ change if the point $P(x, y)$ moves from $P_0(0, 1)$ a distance of $ds = 0.05$ units in the direction of the vector $\vec{v} = 3\vec{i} + 4\vec{j}$?
b) *(8 points)* Let the point $P(x, y)$ follow the curve $(x(t), y(t)) = (t^2 - 8, t - 1)$. Compute

$$\frac{d}{dt}\Big[f(x(t), y(t)) \Big] \text{ at } t = 3.$$

7. a) *(4 points)* Find the critical point(s) of $f(x, y) = x^2 + y^2 - 12x + 6y$ and classify each critical point as a local maximum, a local minimum or a saddle point.
b) *(8 points)* Find the absolute maximum and minimum values of f when we restrict (x, y) to lie in the region $x^2 + y^2 \leq 5$.

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