201 ... e Review Problems for Quiz II — N. Nahlus

1. Find the points on the surface $2x^2 + y^2 - 4xz - 2 = 0$ where the tangent plane is perpendicular to the xy plane. (Hint: Use 2 equations)

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2. The equation $x^5 - x^2 y^2 + 2yz - 6 = 0$ defines x implicitly as a function of y and z. Find the values of $\partial x / \partial y$ at the point A(1, 1, 3).

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3. Find the maximum possible directional derivative of the function $P(x, y, z) = x^4 - 2y^2 + z^4 + 2z + 50$ at the point $(-1, 2, 2)$.

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4) Investigate (for maxima/minma/saddle) the critical point A (1, 1) of the function

$$f(x, y) = x^7 + y^7 - 7xy + 7.$$

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5) Let $P = f(xz, yz^2)$ where $f(u, v)$ is a differentiable function. **Find** $\partial P / \partial X$ at $(x, y, z) = (1, 1, 1)$ given that $f(1,1) = 2$, $f_u(1, 1) = 5$, and $f_v(1,1) = 3$.

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6) (a) Show that $\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^6 y^5}{x^{10} + y^2}\right)$ exists

6) (b). Investigate $\lim_{(x,y)\to(0,0)} \frac{x^6 y^{10}}{x^{10} + y^5}$ by using the curves $y = mx^2$; $y^5 = -x^{10} + mx^{26}$.

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7) On the elliptic plate $x^2 + 2y^2 \le 8$, the temperature function is given by

$$T(x, y) = 2x^2 + 2y^2 - 4y + 22$$

Find the hottest and coldest temperatures on the plate

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8) (a) Set up **but do not evaluate** the integral(s) in polar coordinates that represent the area **outside** the circle $r^2 = 4\sqrt{3}$ and **inside** the lemniscate $r^2 = 8\cos 2\theta$

(**Hint**: $\theta = \pi/12$ (why?). Also $8 > 4\sqrt{3}$)

8) (b) Set up **but do not evaluate** the integral(s) in polar coordinates that represent the area **inside** the circle $r^2 = 4\sqrt{3}$ and **outside** the lemniscate $r^2 = 8\cos 2\theta$

(**Hint**: As above. Also break the region carefully)

A) Find the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $2x^2 + +3y^2 + z^2 = 9$ through the point $P(1, -1, 2)$.

B) Suppose $V = abc$. At $t = 1$: $a = 2, b = 3, c = 1$ & the rate of changes of a, b, c are respectively 0.1, 0.3, -0.5. Find the rate of change of V at $t = 1$.

C) At the point P(1,4), *f(x, y)* has a (directional) derivative $4\sqrt{2}$ in thedirection towards M(2,5) and a derivative of $\sqrt{17}$ in the direction towards *N(5, 3)*. Find the maximum possible derivatice of *f(x, y)* at *P*.

(**Hint**: let *(a, b)* be the gradient vector of *f(x, y)* at *P(1,4)*. So you have 2 equations in 2 unknowns.)

D) Find the points on the surface $x^4 - y^2 + 2yz = 9$ where the tangent plane is parallel to the *x-y* plane. (**Hint**: ∇f is orthogonal to the vector $(0, 0, 1)$ & do NOT forget "Points on surface")

(ANSWER: the points (0, 3, 3) & (0, -3, -3))

E) Let $w = f(x, y)$, $x = r\cos\theta$ & $y = r\sin\theta$. Show that $(f_x)^2 + (f_y)^2 = (w_r)^2 + r^{-2}(w_\theta)^2$

(**Hint**: Compute the right hand - side)