



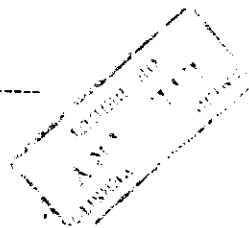
1. Find the points on the surface $2x^2 + y^2 - 4xz - 2 = 0$ where the tangent plane is perpendicular to the xy plane. (Hint: Use 2 equations)

2. The equation $x^5 - x^2y^2 + 2yz - 6 = 0$ defines x implicitly as a function of y and z . Find the values of $\partial x / \partial y$ at the point $A(1, 1, 3)$.

3. Find the maximum possible directional derivative of the function $P(x, y, z) = x^4 - 2y^2 + z^4 + 2z + 50$ at the point $(-1, 2, 2)$.

4. Investigate (for maxima/minima/saddle) the critical point $A(1, 1)$ of the function $f(x, y) = x^7 + y^7 - 7xy + 7$.

5. Let $P = f(xz, yz^2)$ where $f(u, v)$ is a differentiable function. Find $\partial P / \partial x$ at $(x, y, z) = (1, 1, 1)$ given that $f(1, 1) = 2$, $f_u(1, 1) = 5$, and $f_v(1, 1) = 3$.



- 6) (a) Show that $\lim_{(x,y) \rightarrow (0,0)} \cos\left(\frac{x^6 y^5}{x^{10} + y^2}\right)$ exists
- 6) (b). Investigate $\lim_{(x,y) \rightarrow (0,0)} \frac{x^6 y^{10}}{x^{10} + y^5}$ by using the curves $y = mx^2$; $y^5 = -x^{10} + mx^{26}$.

- 7) On the elliptic plate $x^2 + 2y^2 \leq 8$, the temperature function is given by $T(x, y) = 2x^2 + 2y^2 - 4y + 22$. Find the hottest and coldest temperatures on the plate

- 8) (a) Set up **but do not evaluate** the integral(s) in polar coordinates that represent the area **outside** the circle $r^2 = 4\sqrt{3}$ and **inside** the lemniscate $r^2 = 8 \cos 2\theta$ (Hint: $\theta = \pi/12$ (why?). Also $8 > 4\sqrt{3}$)
- 8) (b) Set up **but do not evaluate** the integral(s) in polar coordinates that represent the area **inside** the circle $r^2 = 4\sqrt{3}$ and **outside** the lemniscate $r^2 = 8 \cos 2\theta$ (Hint: As above. Also break the region carefully)

- A) Find the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $2x^2 + 3y^2 + z^2 = 9$ through the point $P(1, -1, 2)$.
- B) Suppose $V = abc$. At $t = 1$: $a = 2, b = 3, c = 1$ & the rate of changes of a, b, c are respectively $0.1, 0.3, -0.5$. Find the rate of change of V at $t = 1$.
- C) At the point $P(1, 4)$, $f(x, y)$ has a (directional) derivative $4\sqrt{2}$ in the direction towards $M(2, 5)$ and a derivative of $\sqrt{17}$ in the direction towards $N(5, 3)$. Find the maximum possible derivative of $f(x, y)$ at P . (Hint: let (a, b) be the gradient vector of $f(x, y)$ at $P(1, 4)$. So you have 2 equations in 2 unknowns.)
- D) Find the points on the surface $x^4 - y^2 + 2yz = 9$ where the tangent plane is parallel to the x - y plane. (Hint: ∇f is orthogonal to the vector $(0, 0, 1)$ & do NOT forget "Points on surface") (ANSWER: the points $(0, 3, 3)$ & $(0, -3, -3)$)
- E) Let $w = f(x, y)$, $x = r \cos \theta$ & $y = r \sin \theta$. Show that $(f_x)^2 + (f_y)^2 = (w_r)^2 + r^{-2}(w_\theta)^2$ (Hint: Compute the right hand - side)