American University of Beirut

MATH 201

Calculus and Analytic Geometry III

Fall 2007

Final Exam

Name: .... ID #: ....

circle your section please: **1 (Th 2-2:50)** **2 (F 2-2:50)** **3 (F 4-4:50)** **4 (F 9-9:50)**

Exercise 1 *(12 points)* Determine if the following series converge or diverge. **Justify your answers**

a. $\displaystyle\sum_{n=0}^{+\infty} \frac{2^{3n}}{1+3^{2n}}$

b. $\displaystyle\sum_{n=1}^{+\infty} \frac{n!}{e^{n^2}}$

c. $\displaystyle\sum_{n=1}^{+\infty} \frac{\ln^n n}{n^2}$

Exercise 2 *(8 points)* Assuming that the following equation

$$2xy + e^{x+y} - 2 = 0$$

defines y as a differentiable function of x, find the value of $\dfrac{dy}{dx}$ at the point $P(0, \ln 2)$

Exercise 3 *(10 points)* Given the surface $z = x^2 - 4xy + y^3 + 4y - 2$ containing the point $P(1, -1, -2)$.

a. Find an equation of the tangent plane to the surface at P.

b. Find an equation of the normal line to the surface at P.

Exercise 4 *(15 points)* Express the volume V under the parabolic cylinder $z = x^2$ above the region enclosed by the parabola $y = 6 - x^2$ and the line $y = x$ in the xy-plane.

a. by using triple integral in the order $dzdydx$

b. by using triple integral in the order $dzdxdy$

c. Evaluate V

Exercise 5 *(15 points)* Convert

$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} 3r\,dz\,dr\,d\theta$$

to **(a)** rectangular coordinates with the order $dzdxdy$ and **(b)** spherical coordinates. Then **(c)** evaluate one of the integrals.

Exercise 6 *(10 points)* Evaluate $\int_0^2 \int_{-\sqrt{1-(y-1)^2}}^0 xy^2 dxdy$ by converting it to polar coordinates

Exercise 7 *(15 points)* Find the absolute minimum and maximum values of

$$f(x, y) = 3xy - 6x - 3y + 7$$

on the closed triangular region R whose vertices are $(0, 0), (3, 0)$ and $(0, 5)$.

Exercise 8 *(20 points)*

a. Find the work done by the force $F = xy\mathbf{i} + (y - x)\mathbf{j}$ along the straight line from $(1, 1)$ to $(2, 3)$

b. **Green's Theorem:** The counter clockwise circulation of a field $F = M\mathbf{i} + N\mathbf{j}$ around a simple closed curve C equals the double integral of (curl $\mathbf{F}$).$\mathbf{k}$ over the region R enclosed by C:

$$\oint_C M dx + N dy = \int\int_R \Big(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\Big) dA \qquad (1)$$

Evaluate

$$\oint_C y^2 dx + 3xydy$$

Where C is the counter clockwise-oriented boundary of the upper-half unit disk.

i. by using the line integral in the left side of equation (1)

ii. *((bonus: 5 points))* by using the double integral in the right side of equation (1)

good luck