American University of Beirut

MATH 201

Calculus and Analytic Geometry III

Spring 2005

Final Exam

Name: ....   ID #: ....

Exercise 1 *[9 points]* Discuss whether the following **series** converges or diverges:

a) $\displaystyle\sum_{n=1}^{+\infty} \frac{\sqrt[n]{n}}{1+n^2}$

b) $\displaystyle\sum_{n=1}^{+\infty} \frac{1.3.5.7...(2n-1)}{(2n)!}$

c) $\displaystyle\sum_{n=0}^{+\infty} \frac{e^n}{3^{n(n+1)}}$

Exercise 2 *[10 points]* Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{\sqrt{n^2+1}} \, (x-\frac{1}{2})^n$

(be sure to check at the end points)

Exercise 3 a) *[5 points]* Find an equation of the tangent plane and a parametric equation for the normal line to the surface $z = 3x^2y - 5x + 1$ at the point $(2, 1, 3)$.

b) *[6 points]* Suppose that $z = e^{xy}$ and that $x = 2u + v, y = u/v$. Use the chain rule to find $\partial z/\partial u$ and $\partial z/\partial v$.

(give the expressions in terms in u and v)

c) *[5 points]* Prove or disprove: $g(x, y) = \dfrac{x^2y^2}{x^4+3y^4}$ can be extended by continuity at $(0, 0)$. Justify.

Exercise 4 *[15 points]* Find the absolute minimum and maximum values of

$$f(x, y) = 3xy - 6x - 3y + 7$$

on the closed triangular region R with vertices $(0, 0), (3, 0)$, and $(0, 5)$.

Exercise 5 *[10 points]* Evaluate the double integral

$$\int_0^2 \int_0^{\sqrt{1-(y-1)^2}} \frac{x}{x^2+y^2} \, dxdy$$

by converting it to a double integral in polar coordinates.

Exercise 6 *[10 points]* Let V be the volume of the tetrahedron bounded by the coordinates planes and the plane $z = 4-4x-2y$. Express V as iterated triple integral in cartesian coordinates in the order:

a) $dxdzdy$ *(do not evaluate the resulting integral).*

b) $dzdydx$, then evaluate the resulting integral.

Exercise 7 *[15 points]* Let V be the volume of the region R cut from the cone $z = \sqrt{x^2+y^2}$ by the two planes $z = 1$ and $z = 2$.

a) Sketch the region of integration.

b) Express V as iterated triple integral in cylindrical coordinates *(do not evaluate the resulting integral).*

c) Express V as iterated triple integral in spherical coordinates, then evaluate the resulting integral.

Exercise 8 *[15 points]*

a) Find the work done by the force field defined by

$$F(x, y, z) = 4y\mathbf{i} + 2xz\mathbf{j} + 3y\mathbf{k}$$

acting on an abject as it moves along the line segment from $(1, 1, 1)$ to $(2, 2, 1)$.

b) Evaluate the line integral $\oint_{(3,-2,0)}^{(1,0,\pi)} (2x\cos z - x^2)dx + (z - 2y)dy + (y - x^2\sin z)dz.$