MATHEMATICS 201
FIRST SEMESTER, 2007-08
QUIZ 1
Solutions

Time: 60 Minutes

Date: November 3, 2007

Name:————————————-

ID Number:——————————

Section:————————————-

Mr. Z. Khachadourian   Prof. A. Lyzzaik

Instructions: The examination consists of two parts: Part I consists of six written questions, and Part II of four multiple-choice questions. Please observe the following:

(a) Answers of questions of Part I must be fully justified.

(b) Circle only one choice (a), (b), $\cdots$, or (e) for each question of Part II.

(c) Each multiple-choice question has exactly one answer.

(d) Circling none or more than one answer for a question of Part II results in a zero credit.

(e) The grade allocated to each question is set next to it.

Part	**Grade**
I	/68
II	/32
TOTAL	/100

PART I: Investigate the following infinite series for convergence and divergence providing adequate justification.

(1) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{\ln n}{n}\right)^3$ (10 points)

Solution. The series converges by the limit comparison test since, by L'Hospital's rule,

$$\lim_{n\to\infty}\frac{(\ln n/n)^3}{(1/n)^2}=\frac{(\ln n)^3}{n}=0,$$

and the $p-$series, $p=2$, $\sum_{n=1}^{\infty}1/n^2$ converges.

(2) $\displaystyle\sum_{n=1}^{\infty}\sqrt[3]{n}\,\sin\left(\frac{1}{\sqrt[3]{n}}\right)$ (10 points)

Solution. The series diverges by the $n-$th term test since

$$\lim_{n\to\infty}\frac{\sin(1/\sqrt[3]{n})}{1/\sqrt[3]{n})}=1.$$

(3) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{n+2}{n+3}\right)^{n^2}$ (10 points)

Solution. The series converges by the $n-$th root test since

$$\begin{aligned}\lim_{n\to\infty}\sqrt[n]{\left(\frac{n+2}{n+3}\right)^{n^2}} &= \lim_{n\to\infty}\left(\frac{n+2}{n+3}\right)^{n}\\ &= \lim_{n\to\infty}\frac{(1+2/n)^n}{(1+3/n)^n}\\ &= e^2/e^3=1/e<1.\end{aligned}$$

(4) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n\ln n}{n^{10}}$ (10 points)

Solution. The series diverges by the ratio test since

$$\begin{aligned}\lim_{n\to\infty}\frac{\dfrac{2^{n+1}\ln(n+1)}{(n+1)^{10}}}{\left(\dfrac{2^n\ln n}{n^{10}}\right)} &= 2\left(\frac{n}{n+1}\right)^{10}\frac{\ln(n+1)}{\ln n}\\ &= 2>1.\end{aligned}$$

(5) $\displaystyle\sum_{n=1}^{\infty} \frac{2.\,4.\,6.\quad\cdots\quad.(2n)}{1.\,3.\,5.\cdots.(2n-1)}.\frac{1}{n}$ (10 points)

Solution. The series diverges by the direct comparison test since

$$\frac{2.\,4.\,6.\quad\cdots\quad.(2n)}{1.\,3.\,5.\cdots.(2n-1)}.\frac{1}{n} = \left(\frac{2}{1}\right)\left(\frac{4}{3}\right)\left(\frac{6}{5}\right)\cdots\left(\frac{2n}{2n-1}\right).\frac{1}{n} > .\frac{1}{n}$$

and the harmonic series diverges.

(6) Given the power series

$$\sum_{n=1}^{\infty}(-1)^n \frac{(x+2)^n}{3^n\ n\ \ln n}.$$

For what values of x does the series converge (i) absolutely and (ii) conditionally? Find the radius of convergence. (18 points)

Solution. Since

$$\begin{aligned}\lim_{n\to\infty}\left|\frac{(-1)^{n+1}\frac{(x+2)^{n+1}}{3^{n+1}\ (n+1)\ \ln(n+1)}}{(-1)^n \frac{(x+2)^n}{3^n\ n\ \ln n}}\right| &= \frac{|x+2|}{3}\lim\frac{n}{n+1}\lim\frac{\ln n}{\ln(n+1)} \\ &= \frac{|x+2|}{3},\end{aligned}$$

by the ratio test for absolute convergence, the series converges absolutely for values of x, $|x+2|<3$, or, $-5<x<1$, and diverges for values of x, $|x+2|>3$, or, $x>1$ or $x<-5$.

If $x=1$, then the series becomes

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{n\ \ln n}$$

which converges by the alternating series test since the sequence $\{1/(n\ln n)\}$ of positive terms is strictly decreasing to zero, but diverges absolutely since the series $\sum 1/(nlnn)$ is a $p-$logarithmic series with $p=1$. Hence, the power series converges conditionally at $x=1$.

If $x=-5$, then the series becomes

$$\sum_{n=1}^{\infty}\frac{1}{n\ \ln n}$$

which diverges as argued above.

PART II: Circle the correct answer in the following multiple-choice questions: (**8 points for each question**)

(7) If the series

$$\sum_{n=0}^{\infty}\frac{(-1)^n\ 2^n}{(n!)^2}$$

is approximated by the sum of its first three terms, then the magnitude of the error made (according to the alternating series estimation theorem) is less than

(a) 1/9
$\rightarrow$ (b) 2/9
(c) 3/4
(d) 2/19
(e) none of the above.

(8) The sum of the series

$$\sum_{n=0}^{\infty} \frac{3^n(n!-1)}{4^n\, n!} \qquad \text{is}$$

(a) $5/2 - e^{3/5}$
(b) $3 - e^{3/4}$
(c) $4 - e^{-3/4}$
$\rightarrow$ (d) $4 - e^{3/4}$
(e) none of the above.

(9) The approximate value with an error of magnitude less than 10^{-3} of the integral

$$\int_0^1 \frac{\sin(x^3)}{x^2}\, dx \qquad \text{is}$$

(a) 15/64
(b) 17/120
$\rightarrow$ (c) 23/48

(d) 23/96
(e) none of the above.

(10) If $f(x) = x^2 e^{-2x}$, then $f^5(0)$ equals

→ (a) -160
(b) -120
(c) 80
(d) -540
(e) none of the above.

[Hint: It is time consuming to attempt successive derivatives]