AMERICAN UNIVERSITY OF BEIRUT
DEPARTMENT OF MATHEMATICS
FALL SEMESTER 2004-05
MATH 201, QUIZ II

Time: 90 minutes

Answer the following questions:

1. Consider the polar curves $r = 1 + 2\cos\theta$ and $r = 4\cos\theta$.
   (a) Sketch the graphs of the polar curves. (8 points)
   (b) Find their points of intersection. (8 points)
   (c) Find the area of the region that lies inside the curve $r = 4\cos\theta$ and outside the curve $r = 1 + 2\cos\theta$. (9 points)

2. Let $f(x) = x/2$ if $-\pi \le x \le \pi$.
   (a) Show that the Fourier series of f is $\sum_{n=1}^{\infty}(-1)^{n+1}(\sin nx)/n$. (9 points)
   (b) For what values of x, $-\pi \le x \le \pi$, does the equality

$$\frac{x}{2} = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$$

hold? Justify your answer. (8 points)
   (c) Use (b) to conclude the value of $\pi/4$ as an alternating series. (8 points)

3. (a) By discussing the limit of the function

$$f(x, y) = \frac{x^4 + y^4}{(x^2 + y^2)^{3/2}} \quad ((x, y) \ne (0, 0))$$

at $(0, 0)$, decide whether $f(0, 0)$ can be defined so that f becomes continuous at $(0, 0)$. (8 points)
   (b) Find an equation for the tangent plane to the paraboloid $z = 2x^2 + 3y^2$ at the point $P(1, -1, 5)$. (8 points)
   (c) Find and classify the critical points of the function $f(x, y) = 2x^3 - 3x^2 + y^2 - 12x + 10$. (9 points)

4. (a) The temperature T at (x, y, z) is given by

$$T(x, y, z) = 4x^2 - y^2 + 16z^2.$$

Find the maximum rate of change of T at the point $P(4, 2, -1)$ and the direction in which it is achieved. (9 points)
   (b) Find an equation for the normal line to the curve $2x^3 + x^2y = 1 - y^3$ at the point $P(-1/2, 1)$. (8 points)
   (c) Sketch the level surface passing through the point $P(1, 1, 1)$ of the function $f(x, y, z) = x^2 + y^2 - z^2$. (8 points)

GOOD LUCK